1. **HONOR CODE (1 point)**
   Please copy the following statements into the box provided for the honor code on your answer sheet, and sign your name.

   *As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.*

2. **What’s your favorite hobby? (1 point) All answers will be awarded full credit.**
3. How vast do they span? (5 points)

(a) (3 points) Let $S$ be the following set of vectors from $\mathbb{R}^3$:

$$S = \left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix} \right\}$$

Choose the minimum number of vectors from the list below to add to $S$ so that it spans $\mathbb{R}^3$. Explain your choice.

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 3 \\ 9 \\ 6 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

**Solution:** Set $S$ contains \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, and \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}, a scaled multiple of the first vector and therefore linearly dependent. Thus, we need to add at least two more linearly independent vectors to $S$ so that it spans $\mathbb{R}^3$.

In the list of vectors, $\vec{v}_2$ is also a scaled version of \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} while $\vec{v}_1$ and $\vec{v}_3$ are not. Performing Gaussian Elimination on the set $S = \left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \end{bmatrix} \right\}$ gives 3 pivots/linearly independent columns, meaning that they spans $\mathbb{R}^3$.

(b) (2 points) Does $S$, with the vectors you added from the previous part, form a basis of $\mathbb{R}^3$? Explain.

**Solution:** Although $S$ spans $\mathbb{R}^3$, it contains more than 3 vectors. Since a basis is a minimum set of spanning vectors, $S$ is not a basis.
4. The Solution to the Solution (8 points)

(a) (4 points) Rocky is trying to solve a system of equations. He writes out the following augmented matrix for the system he is trying to solve:

\[
\begin{bmatrix}
1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8 \\
9 & 10 & 11 & \alpha
\end{bmatrix}
\]

What values of \( \alpha \) allow for no solutions, one solution and infinite solutions respectively? If no values of \( \alpha \) are possible, write N/A. Write your answers in the blanks below.

Solution: No solutions: \( \alpha \neq 12 \)
One Solution: N/A
Infinite Solutions: \( \alpha = 12 \)
First, we will try to solve the augmented matrix to see what we get:

\[
\begin{bmatrix}
1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8 \\
9 & 10 & 11 & \alpha
\end{bmatrix}
\sim
\begin{bmatrix}
1 & 2 & 3 & 4 \\
0 & -4 & -8 & -12 \\
0 & 10 & 11 & \alpha
\end{bmatrix}
\sim
\begin{bmatrix}
1 & 2 & 3 & 4 \\
0 & -4 & -8 & -12 \\
0 & 0 & 36 - \alpha & \alpha - 36
\end{bmatrix}
\sim
\begin{bmatrix}
1 & 2 & 3 & 4 \\
0 & 1 & 2 & \frac{36 - \alpha}{8} \\
0 & 0 & 0 & \frac{36 - \alpha}{8} - 3
\end{bmatrix}
\]

In row echelon form, we see that if \( \frac{36 - \alpha}{8} - 3 = 0 \), then there are two pivots, which means there are infinite solutions. If \( \frac{36 - \alpha}{8} - 3 \neq 0 \), then we do not have a solution as the augmented matrix would be inconsistent. Solving the equation, we see that \( \alpha = 12 \) leads to infinite solutions, and \( \alpha \neq 12 \) gives no solutions. There is no case that leads to only one solution.

(b) (4 points) Rolly tries to copy Rocky’s matrix, but accidentally copies it incorrectly by tranposing it. Instead, she gets the following augmented matrix:

\[
\begin{bmatrix}
1 & 5 & 9 \\
2 & 6 & 10 \\
3 & 7 & 11 \\
4 & 8 & \alpha
\end{bmatrix}
\]

What values of \( \alpha \) allow for no solutions, one solution and infinite solutions respectively? If no values of \( \alpha \) are possible, write N/A. Write your answers in the blanks below.

Solution: No solutions: N/A
One Solution: N/A
Infinite Solutions: \( \alpha = \frac{36 - \alpha}{8} \)}

**Solution:**  
No solutions: $\alpha \neq 12$  
One Solution: $\alpha = 12$  
Infinite Solutions: N/A

Similar to part a, we will try to solve the augmented matrix to see what we get:

\[
\begin{bmatrix}
1 & 5 & 9 \\
2 & 6 & 10 \\
3 & 7 & 11 \\
4 & 8 & \alpha \\
\end{bmatrix}
\sim
\begin{bmatrix}
1 & 5 & 9 \\
0 & -4 & -8 \\
0 & -8 & -16 \\
0 & -12 & \alpha - 24 \\
\end{bmatrix}
\]

\[
\sim
\begin{bmatrix}
1 & 5 & 9 \\
0 & 1 & 2 \\
0 & 1 & 2 \\
0 & 1 & \frac{24 - \alpha}{12} \\
\end{bmatrix}
\]

\[
\sim
\begin{bmatrix}
1 & 5 & 9 \\
0 & 1 & 2 \\
0 & 0 & 0 \\
0 & 0 & \frac{24 - \alpha}{12} - 2 \\
\end{bmatrix}
\]

In row echelon form, we see that if $\frac{24 - \alpha}{12} - 2 = 0$, then there are two pivots, which is exactly enough for one solution. If $\frac{36 - \alpha}{8} - 3 \neq 0$, then we do not have a solution as the augmented matrix would be inconsistent. Solving the equation, we see that $\alpha = 12$ leads to one solution, and $\alpha \neq 12$ gives no solutions. There is no case that leads to infinite solutions.
5. **A good enough basis (3 points)**

Let $S$ be the following set of vectors from $\mathbb{R}^6$:

$$S = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

Show that $S$ is linearly dependent.

**Solution:** To show linear dependence of a set of vectors, it suffice to find a set of non-all-zero coefficients so that the linear combinations of vectors is the zero vector: $\vec{v}_1 - \vec{v}_2 + \vec{v}_3 - \vec{v}_4 + \vec{v}_5 - \vec{v}_6 = \vec{0}$
6. Eigen-Sleuths (5 points)

Martin has a matrix $A$ that has special properties for his project, but he forgot what it was! All he knows is that the matrix fulfills the special condition below:

$$A^2 = 4I$$

where $I$ is the identity matrix. Luckily, he only needs eigenvalues of the matrix in his project. Find all the possible eigenvalues of $A$. Show your work.

**Solution:** Assume that $x$ is an arbitrary eigenvector of $A$ corresponding with the eigenvalue $\lambda$. Then if we multiply both sides of the equation above by $x$, we get:

$$A^2 x = 4Ix$$

$$A^2 x = 4x$$

$$A\lambda x = 4x$$

$$\lambda^2 x = 4x$$

$$(\lambda^2 - 4)x = 0$$

Since $x$ is nonzero by definition of an eigenvector, this means that $\lambda^2 - 4 = 0$ in order for this equation to be true. This breaks down into $(\lambda - 2)(\lambda + 2) = 0 \implies \lambda = -2, 2$. Thus, the only two possible eigenvalues of $A$ are -2 and 2.

Note: If you stated that $A = 2I$, this would not give an eigenvalue of -2. Not all matrices will have all the possible eigenvalues even if it fulfills this condition.
7. Give me my Space (5 points)

Based on the following information about matrix $M$ answer the questions that follow: The matrix $M$ transforms $\vec{v}_1$ to $M\vec{v}_1$ as shown in the graph below. The matrix $M$ also transforms $\vec{v}_2$ to $M\vec{v}_2$ as shown in the graph below.

(a) (1 points) What is the shape of the matrix $M$?

**Solution:** Let the matrix $M$ be an $m \times n$ matrix. Since we can multiply it with a 2D vector, $n=2$, and since the vector resulting from this multiplication is also a 2D vector, $m=2$. So $M$ must be a $2 \times 2$ matrix.
(b) (2 points) What is the Column Space of $M$? What is the dimension of the Column Space of $M$?

**Solution:** From the graph we can see that the matrix $M$ maps $\vec{v}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ to $M\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and similarly the matrix maps $\vec{v}_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ to $M\vec{v}_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

Any 2D vector can be expressed as $\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix} = (-x) \begin{bmatrix} -1 \\ 0 \end{bmatrix} + y \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -x\vec{v}_2 + y\vec{v}_1$

So, $M\vec{v} = M(-x\vec{v}_2 + y\vec{v}_1) = -xM\vec{v}_2 + yM\vec{v}_1 = -x\begin{bmatrix} -1 \\ -1 \end{bmatrix} + y \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} x+y \\ x+y \end{bmatrix} = (x+y) \begin{bmatrix} 1 \\ 1 \end{bmatrix} \in \text{span} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Hence the Column Space of $M$ is $\text{span} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, since the space is the span of a single vector, it is a one dimensional space.

(c) (2 points) What is the Null Space of $M$? What is the dimension of the Null Space of $M$?

**Solution:** The Null Space is the set of vectors which get mapped to the zero vector by the matrix $M$.

In part (b) we saw that for any 2D vector $\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix}$ $M\vec{v} = \begin{bmatrix} x+y \\ x+y \end{bmatrix}$

From the above result we can conclude that for any vector to be mapped to the zero vector $x + y = 0$ or $x = -y$. Every vector that satisfies this condition is in the Null Space of $M$.

$\vec{v}_{null} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ y \end{bmatrix} = y \begin{bmatrix} -1 \\ 1 \end{bmatrix} \in \text{span} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Hence the Null Space of $M$ is $\text{span} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, since the space is the span of a single vector, it is a one dimensional space.
8. Favorite Study Spots in Berkeley (8 points)

Berkeley students are some of the most studious in the nation! Thus, it is not uncommon to find them studying in various spots on campus. Due to class schedules, students often move to different libraries depending on their proximity to the different classes. The flow of students across the four most popular libraries is as follows:

Let the number of students at the libraries be represented in the following way:

\[
A = \begin{bmatrix}
  x_{ML}[t]
  x_{DL}[t]
  x_{MS}[t]
  x_{EAL}[t]
\end{bmatrix} = \begin{bmatrix}
  x_{ML}[t+1]
  x_{DL}[t+1]
  x_{MS}[t+1]
  x_{EAL}[t+1]
\end{bmatrix}
\]

(a) (2 points) Write the transition matrix \(A\) corresponding to the diagram above. Solution:

\[
\begin{bmatrix}
  0.5 & 0.1 & 0.1 & 0.2 \\
  0 & 0.4 & 0 & 0 \\
  0.2 & 0.4 & 0.6 & 0 \\
  0.3 & 0.1 & 0 & 0.5
\end{bmatrix}
\]
(b) (2 points) Determine if the transition matrix is conservative or not. Explain why or why not either conceptually or mathematically.

Solution:
The definition of a conservative transition matrix implies that its columns must add up to 1. This is not the case for the third and fourth columns of the matrix; thus, we can derive that the matrix is not conservative.

(c) (1 points) For a research project, your friend wants to predict the number of students studying in these libraries in the future. Ignoring your answer from the previous part, use the following transition matrix $A$, and the current number of students given by $\vec{x}[t]$:

$$A = \begin{bmatrix}
0.5 & 0.4 & 0 & 0.3 \\
0.3 & 0.4 & 0.5 & 0 \\
0 & 0 & 0.2 & 0.2 \\
0.2 & 0.2 & 0.3 & 0.5
\end{bmatrix} \quad \vec{x}[t] = \begin{bmatrix}
140 \\
400 \\
210 \\
90
\end{bmatrix}$$

Help your friend predict the number of students in each libraries in the next time step.

Solution:

$$\vec{x}[t + 1] = A\vec{x}[t] = \begin{bmatrix}
257 \\
307 \\
60 \\
216
\end{bmatrix}$$
(d) (3 points) You want to expand upon your friend’s research. Thus, you tracked the number of students at 2 lesser known libraries (Hangrove Library and Mathematics/Statistics Library) and calculated their corresponding state-transition matrix to form the following model:

\[
\begin{bmatrix}
0.7 & 0.6 \\
0.3 & 0.4
\end{bmatrix}
\begin{bmatrix}
x_{H}[t] \\
x_{MS}[t]
\end{bmatrix}
=
\begin{bmatrix}
x_{H}[t+1] \\
x_{MS}[t+1]
\end{bmatrix}
\]

Given that there are in total 1500 students, determine the number of students in these two libraries after infinite time steps ($\vec{x}[\infty]$). If the answer can not be determined, give a brief explanation why.

**Solution:** We see that $0.7 + 0.3 = 1$ and $0.6 + 0.4 = 1$; thus, we know that $A$ is conservative, meaning $\vec{x}[\infty]$ exists. To find the steady-state, we need to calculate the eigenvector of the transition matrix that corresponds to an eigenvalue of 1.

\[
(A - \lambda I)\vec{x} = 0
\]

\[
\begin{bmatrix}
-0.3 & 0.6 \\
0.3 & -0.6
\end{bmatrix}
\begin{bmatrix}
\vec{x} \\
\vec{0}
\end{bmatrix}
=
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

\[
\vec{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}
\]

Make sure to scale the eigenvector so that the sum of the elements correspond to the total number of students:

\[
\vec{x}[\infty] = \begin{bmatrix} 1000 \\ 500 \end{bmatrix}
\]
9. Triforce Equivalence (9 points)

Each resistor in the following circuits has a resistance $R$. Find the equivalent resistance between nodes $a$ and $b$ in for each circuit (Leave your final answer in terms of $R$):

(a) (3 points)

Solution: On close observation you can see that nodes $a$ and $b$ are shorted by the 2 diagonal wires which meet at the top in the circuit. Hence, for this part: $R_{ab} = 0$

(b) (3 points)

Solution:

Here we can see that there are 3 nodes in the circuit $a$, $b$, $c$. So we can redraw the circuit to simplify our analysis. There are 2 resistors from $a$-$c$, 2 from $a$-$b$ and one from $c$-$b$. 
\[ R_{ab} = \frac{3R}{8} \]
(c) (3 points)
[Hint: Think about the "interesting circuit" we discussed in class.]

Solution:

The Five colored resistors form the ‘interesting circuit’ discussed in lecture (also called a balanced Wheatstone bridge). We know that no current flows through the red resistor. Hence we can ignore that resistor to find the equivalent resistance between nodes a and b. The new equivalent circuit to analyse would be:

\[ R_{ab} = (R + R)||(R + R)||R = \frac{R}{2} \]
10. The matrix of a circuit (8 points)

Consider the following circuit:

(a) (4 points) Write the KCL expressions for all nodes with unknown voltages.

**Solution:** The signs of the elemental currents can vary based on the labeling, one possible answer is:

\[
\begin{align*}
I_1 &= I_2 + I_4, \\
I_2 &= I_3, \\
I_3 + I_4 &= I_5, \\
I_5 &= I_6
\end{align*}
\]
(b) (4 points) Given $V_S = 8 \text{ V}$, $R_1 = 4 \text{ k}\Omega$, $R_2 = 4 \text{ k}\Omega$, $R_3 = 2 \text{ k}\Omega$, $R_4 = 2 \text{ k}\Omega$, $R_5 = 1 \text{ k}\Omega$, $R_6 = 4 \text{ k}\Omega$, find the unknown entries of the matrix below that solves for the nodal voltages.

$$
\begin{bmatrix}
 a & b & c & d \\
 e & f & g & h \\
 i & j & k & l \\
 m & n & o & p \\
\end{bmatrix}
\begin{bmatrix}
 u_1 \\
 u_2 \\
 u_3 \\
 u_4 \\
\end{bmatrix}
= 
\begin{bmatrix}
 2 \\
 0 \\
 0 \\
 0 \\
\end{bmatrix}
$$

**Solution:** From part (a) we have the KCL equations

$$
I_1 = I_2 + I_4,
I_2 = I_3,
I_3 + I_4 = I_5,
I_5 = I_6
$$

Express the elemental voltages in terms of node voltages:

$$
V_1 = V_S - u_1,
V_2 = u_1 - u_2,
V_3 = u_2 - u_3,
V_4 = u_1 - u_3,
V_5 = u_3 - u_4,
V_6 = u_4 - 0 = u_4
$$

According to Ohm’s Law, we know:

$$
I_1 = \frac{V_1}{R_1} = \frac{V_S - u_1}{R_1},
I_2 = \frac{V_2}{R_2} = \frac{u_1 - u_2}{R_2},
I_3 = \frac{V_3}{R_3} = \frac{u_2 - u_3}{R_3},
I_4 = \frac{V_4}{R_4} = \frac{u_1 - u_3}{R_4},
I_5 = \frac{V_5}{R_5} = \frac{u_3 - u_4}{R_5},
I_6 = \frac{V_6}{R_6} = \frac{u_4}{R_6},
$$

Plug into the KCL expression:

$$
\frac{V_S - u_1}{R_1} = \frac{u_1 - u_2}{R_2} + \frac{u_1 - u_3}{R_4},
\frac{u_1 - u_2}{R_2} = \frac{u_2 - u_3}{R_3},
\frac{u_2 - u_3}{R_3} + \frac{u_1 - u_3}{R_4} = \frac{u_3 - u_4}{R_5},
\frac{u_3 - u_4}{R_5} = \frac{u_4}{R_6}
$$
Plug in the given values of the resistors and the voltage source and simplify (note that the resistors we were given were in $k\Omega$s, but we can just multiply each equation above by 1000 to covert to $\Omega$s):

\[
\begin{align*}
    u_1 - \frac{1}{4} u_2 - \frac{1}{2} u_3 &= 2, \\
    \frac{1}{4} u_1 - \frac{3}{4} u_2 + \frac{1}{2} u_3 &= 0, \\
    \frac{1}{2} u_1 + \frac{1}{2} u_2 - 2 u_3 + u_4 &= 0, \\
    u_3 - \frac{5}{4} u_4 &= 0
\end{align*}
\]

Rewrite the system of linear equations in matrix-vector form to obtain the coefficients:

\[
\begin{bmatrix}
    1 & -\frac{1}{4} & -\frac{1}{2} & 0 \\
    \frac{1}{4} & -\frac{3}{4} & \frac{1}{2} & 0 \\
    \frac{1}{2} & \frac{1}{2} & -2 & 1 \\
    0 & 0 & 1 & -\frac{5}{4}
\end{bmatrix}
\begin{bmatrix}
    u_1 \\
    u_2 \\
    u_3 \\
    u_4
\end{bmatrix}
=
\begin{bmatrix}
    2 \\
    0 \\
    0 \\
    0
\end{bmatrix}
\]
11. Resistive Touchscreen (9 points)

Let $R_1 = R_2 = R_5 = R_6 = R_9 = R_{10} = R_{11} = R_{12} = 50\Omega$, $R_3 = R_4 = R_7 = R_8 = 100\Omega$, and $V_s = 1V$

(a) (2 points) What is the voltage at $V_{mid}$ (in other words, what is the voltage across $R_8$)?

**Solution:** No current flows through $R_3$ and $R_4$, simplifying our problem significantly. This means whatever voltage drop across $R_8$ would be a single voltage divider.

$$V_{mid} = V_s \frac{R_8}{R_8 + R_7} = \frac{1}{2} V$$
(b) (2 points) What is the current through $R_{11}$?

**Solution:** Again, no current flows through $R_3$ and $R_4$, simplifying our problem significantly. We can solve for the current through $R_{11}$ in multiple ways: we can do KCL or we can just use Ohm’s law! Since no current flows through $R_3$, we know that whatever current enters the branch (through $R_1$) must also flow out of the branch (through $R_{12}$). This means whatever current flows through $R_1$ also flows through $R_2$ and $R_{11}$.

$$I_{R_{11}} = \frac{V_s}{R_1 + R_2 + R_{11} + R_{12}} = \frac{1}{200} A \tag{5}$$

(c) (3 points) What is the total power dissipated by this touch screen?

**Solution:** Power is defined as the total current through a circuit times the total voltage drop across a circuit. We already know the total voltage drop ($V_s$), we just need to find the total current flowing through the touch screen. Luckily we already calculated the current flowing through a single branch, we just need to find the current through the other branches and sum them all together. Eagle eye’d viewers would notice that each branch has the same impedance going from top to bottom so we don’t have to do any further work (you could repeat the same steps from part b to convince yourself if you’d like - but I’m not - I’m already convinced). So we can just multiply the current through $R_{11}$ by 3 and call it a day:

$$P = V_s \times (3 \times I_{R_{11}}) = \frac{3}{200} W \tag{6}$$
(d) (2 points) Imagine that $R_4$ breaks and becomes an open circuit. Would $V_{mid}$ change? Would this mean the touch screen is broken?

**Solution:** No current flows through $R_4$ so $V_{mid}$ doesn’t actually change. No. The touch screen would still work!
12. A new superhero: Superposition (8 points)

\[ R_1 = R_2 = R_3 = R_4 = 1k\Omega. \] Find the current through each resistor.

**Solution:** The circuit is the superposition of 4 circuits:

Here we can see that \( i_2 = -6A, \quad i_1 = i_3 = i_4 = 2A \) using the current divider expression.
Here we can see that $i_4 = -3A$, $i_1 = i_2 = i_3 = 1A$ using the current divider expression.

Here we can see that $i_1 = -6A$, $i_2 = i_3 = i_4 = 2A$ using the current divider expression.
Here we can see that $i_3 = -3A$, $i_1 = i_2 = i_4 = 1A$ using the current divider expression.

The final currents are the sum of currents because of each source:

- $i_1 = -2A$
- $i_2 = -2A$
- $i_3 = 2A$
- $i_4 = 2A$
13. No Cap (5 points)

(a) (3 points) Find the equivalent capacitance between nodes A and B in terms of each of the capacitors, \( C_i \). You can use the parallel operator (\( \parallel \)) for simplification. NOTE: \( c_1 || c_2 = \frac{c_1 c_2}{c_1 + c_2} \)

\[
C_{eq} = \left( \left( \left( C_4 + C_7 \right) \parallel \left( C_1 \parallel C_2 \right) \right) + C_3 \right) \parallel C_6
\]

Solution: Notice that \( C_5 \) is shorted. The equivalent circuit is:
(b) (2 points) Unfortunately, when trying to physically reproduce this circuit by replacing it with the equivalent capacitor $C_{eq}$, you realize you don’t have an $0.8 \times 10^{-6} F$ capacitor! With your EECS16A knowledge, you decide to build one.

You have at your disposition, on top of two metal plates of length $l_m = 200 \text{ cm}$, width $w_m = 50 \text{ cm}$:

- a piece of silicon, $\varepsilon_s = 12 \times 10^{-12} \frac{F}{m}$;
- a piece of glass, $\varepsilon_g = 4 \times 10^{-12} \frac{F}{m}$;
- a piece of wood, $\varepsilon_w = 2 \times 10^{-12} \frac{F}{m}$;

of same dimensions with a thickness of $t = 5 \times 10^{-6} m$, length $l_i = 300 \times 10^{-2} m$, width $w_i = 60 \times 10^{-2} m$.

Which of the materials above should you place between the two metal plates to build yourself a capacitor of value $0.8 \times 10^{-6} F$?

**Solution:**

\[ C = \varepsilon \frac{A}{d} \]

The distance between the metal plates is determined by the thickness of the material, then $d = 5 \times 10^{-6} m$.

Similarly, $A$ is determined by the area between the two plates. Then,

\[ A = l_m \times w_m \]

\[ A = 1m^2 \]

Since,

\[ \varepsilon = \frac{C \times d}{A} \]

Then,

\[ \varepsilon = 0.8 \times 10^{-6} \times 5 \times 10^{-6} \]

\[ \varepsilon = 4 \times 10^{-12} \frac{F}{m} \]

Which corresponds to the glass piece.