1. HONOR CODE (1 point)
Please copy the following statements into the box provided for the honor code on your answer sheet, and sign your name.

As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.

2. What’s your favorite hobby? (1 point) All answers will be awarded full credit.
3. How vast do they span? (5 points)

(a) (3 points) Let $S$ be the following set of vectors from $\mathbb{R}^3$:

$$S = \left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix} \right\}$$

Choose the minimum number of vectors from the list below to add to $S$ so that it spans $\mathbb{R}^3$. Explain your choice.

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 3 \\ 9 \\ 6 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 5 \\ 1 \\ 7 \end{bmatrix}$$

(b) (2 points) Does $S$, with the vectors you added from the previous part, form a basis of $\mathbb{R}^3$? Explain.
4. The Solution to the Solution (8 points)

(a) (4 points) Rocky is trying to solve a system of equations. He writes out the following augmented matrix for the system he is trying to solve:

\[
\begin{bmatrix}
1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8 \\
9 & 10 & 11 & \alpha \\
\end{bmatrix}
\]

What values of \( \alpha \) allow for no solutions, one solution and infinite solutions respectively? If no values of \( \alpha \) are possible, write N/A. Write your answers in the blanks below.
(b) (4 points) Rolly tries to copy Rocky’s matrix, but accidentally copies it incorrectly by tranposing it. Instead, she gets the following augmented matrix:

\[
\begin{bmatrix}
1 & 5 & 9 \\
2 & 6 & 10 \\
3 & 7 & 11 \\
4 & 8 & \alpha
\end{bmatrix}
\]

What values of \( \alpha \) allow for no solutions, one solution and infinite solutions respectively? If no values of \( \alpha \) are possible, write N/A. Write your answers in the blanks below.
5. A good enough basis (3 points)

Let $S$ be the following set of vectors from $\mathbb{R}^6$:

$$S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

Show that $S$ is linearly dependent.
6. **Eigen-Sleuths (5 points)**

Martin has a matrix $A$ that has special properties for his project, but he forgot what it was! All he knows is that the matrix fulfills the special condition below:

$$A^2 = 4I$$

where $I$ is the identity matrix. Luckily, he only needs eigenvalues of the matrix in his project. Find all the possible eigenvalues of $A$. Show your work.
7. **Give me my Space (5 points)**

Based on the following information about matrix $M$ answer the questions that follow: The matrix $M$ transforms $\vec{v}_1$ to $M\vec{v}_1$ as shown in the graph below. The matrix $M$ also transforms $\vec{v}_2$ to $M\vec{v}_2$ as shown in the graph below.

(a) (1 points) What is the shape of the matrix $M$?
(b) (2 points) What is the Column Space of $M$? What is the dimension of the Column Space of $M$?

(c) (2 points) What is the Null Space of $M$? What is the dimension of the Null Space of $M$?
8. Favorite Study Spots in Berkeley (8 points)

Berkeley students are some of the most studious in the nation! Thus, it is not uncommon to find them studying in various spots on campus. Due to class schedules, students often move to different libraries depending on their proximity to the different classes. The flow of students across the four most popular libraries is as follows:

Let the number of students at the libraries be represented in the following way:

\[
\begin{bmatrix}
    x_{ML}[t] \\
    x_{DL}[t] \\
    x_{MS}[t] \\
    x_{EAL}[t]
\end{bmatrix}
= \begin{bmatrix}
    x_{ML}[t + 1] \\
    x_{DL}[t + 1] \\
    x_{MS}[t + 1] \\
    x_{EAL}[t + 1]
\end{bmatrix}
\]

(a) (2 points) Write the transition matrix \( A \) corresponding to the diagram above.
(b) (2 points) Determine if the transition matrix is conservative or not. Explain why or why not either conceptually or mathematically.

(c) (1 points) For a research project, your friend wants to predict the number of students studying in these libraries in the future. Ignoring your answer from the previous part, use the following transition matrix $A$, and the current number of students given by $\vec{x}[t]$:

$$A = \begin{bmatrix}
0.5 & 0.4 & 0 & 0.3 \\
0.3 & 0.4 & 0.5 & 0 \\
0 & 0 & 0.2 & 0.2 \\
0.2 & 0.2 & 0.3 & 0.5
\end{bmatrix}, \quad \vec{x}[t] = \begin{bmatrix} 140 \\ 400 \\ 210 \\ 90 \end{bmatrix}$$

Help your friend predict the number of students in each libraries in the next time step.
(d) (3 points) You want to expand upon your friend’s research. Thus, you tracked the number of students at 2 lesser known libraries (Hangrove Library and Mathematics/Statistics Library) and calculated their corresponding state-transition matrix to form the following model:

\[
\begin{bmatrix}
0.7 & 0.6 \\
0.3 & 0.4
\end{bmatrix}
\begin{bmatrix}
x_{H}[t] \\
x_{MS}[t]
\end{bmatrix}
=
\begin{bmatrix}
x_{H}[t+1] \\
x_{MS}[t+1]
\end{bmatrix}
\]

Given that there are in total 1500 students, determine the number of students in these two libraries after infinite time steps ($\vec{x}_{\infty}$). If the answer can not be determined, give a brief explanation why.
9. Triforce Equivalence (9 points)

Each resistor in the following circuits has a resistance \( R \). Find the equivalent resistance between nodes \( a \) and \( b \) in for each circuit (Leave your final answer in terms of \( R \)):

(a) (3 points)
(b) (3 points)
(c) (3 points)

[Hint: Think about the "interesting circuit" we discussed in class.]
10. The matrix of a circuit (8 points)

Consider the following circuit:

(a) (4 points) **Write the KCL expressions** for all nodes with unknown voltages.
(b) (4 points) Given $V_5 = 8$ V, $R_1 = 4$ kΩ, $R_2 = 4$ kΩ, $R_3 = 2$ kΩ, $R_4 = 2$ kΩ, $R_5 = 1$ kΩ, $R_6 = 4$ kΩ, find the unknown entries of the matrix below that solves for the nodal voltages.

\[
\begin{bmatrix}
 a & b & c & d \\
 e & f & g & h \\
 i & j & k & l \\
 m & n & o & p
\end{bmatrix}
\begin{bmatrix}
 u_1 \\
 u_2 \\
 u_3 \\
 u_4
\end{bmatrix}
= \begin{bmatrix}
 2 \\
 0 \\
 0 \\
 0
\end{bmatrix}
\]
11. Resistive Touchscreen (9 points)

Let $R_1 = R_2 = R_5 = R_6 = R_9 = R_{10} = R_{11} = R_{12} = 50\, \Omega$, $R_3 = R_4 = R_7 = R_8 = 100\, \Omega$, and $V_s = 1V$

(a) (2 points) What is the voltage at $V_{mid}$ (in other words, what is the voltage across $R_8$)?
(b) (2 points) What is the current through $R_{11}$?

(c) (3 points) What is the total power dissipated by this touch screen?
(d) (2 points) Imagine that $R_d$ breaks and becomes an open circuit. Would $V_{\text{mid}}$ change? Would this mean the touch screen is broken?
12. A new superhero: Superposition (8 points)

(a) (10 points) $R_1 = R_2 = R_3 = R_4 = 1k\Omega$. Find the current through each resistor.
13. No Cap (5 points)

(a) (3 points) Find the equivalent capacitance between nodes $A$ and $B$ in terms of each of the capacitors, $C_l$. You can use the parallel operator ($\parallel$) for simplification. NOTE: $c_1 \parallel c_2 = \frac{c_1 c_2}{c_1 + c_2}$
(b) (2 points) Unfortunately, when trying to physically reproduce this circuit by replacing it with the equivalent capacitor $C_{eq}$, you realize you don’t have an $0.8 \times 10^{-6} F$ capacitor! With your EECS16A knowledge, you decide to build one.

You have at your disposition, on top of two metal plates of length $l_m = 200$ cm, width $w_m = 50$ cm:

- a piece of silicon, $\varepsilon_s = 12 \times 10^{-12} \frac{F}{m}$;
- a piece of glass, $\varepsilon_g = 4 \times 10^{-12} \frac{F}{m}$;
- a piece of wood, $\varepsilon_w = 2 \times 10^{-12} \frac{F}{m}$;

of same dimensions with a thickness of $t = 5 \times 10^{-6} m$, length $l_i = 300 \times 10^{-2} m$, width $w_i = 60 \times 10^{-2} m$.

Which of the materials above should you place between the two metal plates to build yourself a capacitor of value $0.8 \times 10^{-6} F$?
PRINT your student ID: 

Extra page for scratchwork.

Work on this page will NOT be graded.