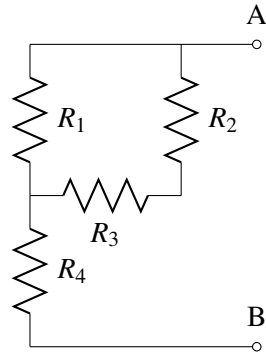


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3. Simplifying Resistor Networks (6 Points)

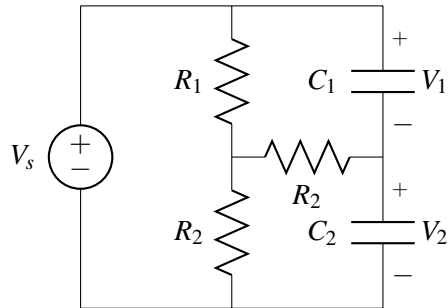
Determine the equivalent resistance between points A and B in the circuit shown below. You may use the parallel operator \parallel in your final answer.



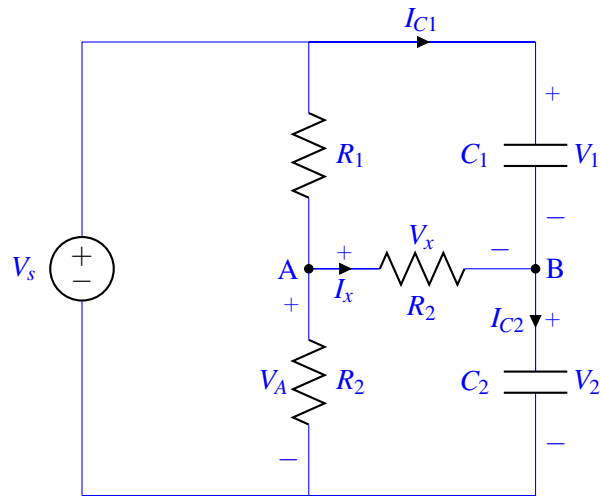
Solutions: $R_{eq} = R_1 \parallel (R_2 + R_3) + R_4$

4. Capacitors (6 Points)

For the circuit given below, determine the voltage across each capacitor at steady state. $R_2 = 2R_1$, $C_2 = C_1$, and $V_s = 3\text{ V}$.



Solutions: We label the circuit as follows:



In steady state,

$$I_{C1} = 0$$

$$I_{C2} = 0$$

By KCL at node B ,

$$I_x = 0$$

which implies

$$V_x = 0$$

$$V_A = V_B$$

Since $I_x = 0$, then by the voltage divider equation.

$$V_A = \frac{R_2}{R_1 + R_2} V_s$$

By KVL:

$$V_2 = V_A - V_x$$

$$= V_A$$

$$= \frac{R_2}{R_1 + R_2} V_s$$

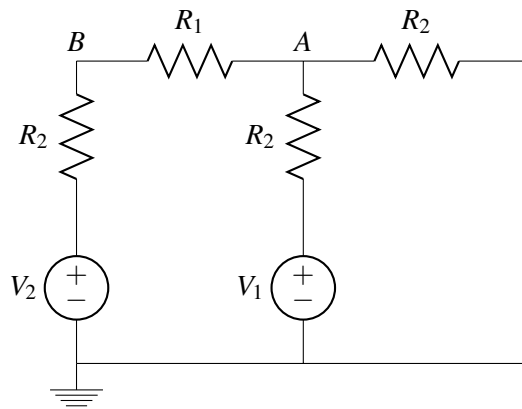
$$V_1 = V_s - V_2$$

$$= \frac{R_1}{R_1 + R_2} V_s$$

Therefore, $V_1 = 1 \text{ V}$ and $V_2 = 2 \text{ V}$.

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5. Ladder Nodal (8 Points)



Use nodal analysis to write equations for each node A and B .

Let $R_1 = 1\Omega$, $R_2 = 2\Omega$, and leave your expressions in terms of V_1 , V_2 , R_1 , R_2 , V_A and V_B . Do not simplify.

Solutions:

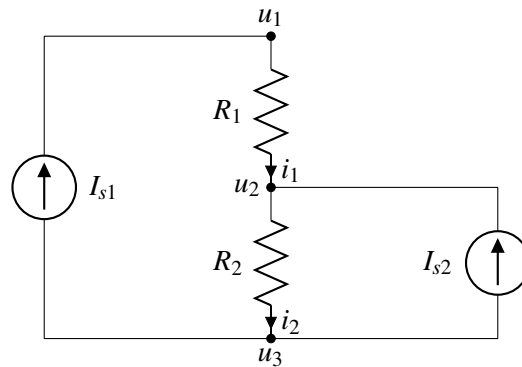
For node A :

$$0 = \frac{V_A}{R_2} + \frac{(V_A - V_1)}{R_2} + \frac{(V_A - V_B)}{R_1}$$
$$\frac{1}{R_2}V_1 = V_A \left(\frac{1}{R_2} + \frac{1}{R_2} + \frac{1}{R_1} \right) - V_B \left(\frac{1}{R_1} \right)$$
$$0.5V_1 = 2V_A - V_B$$

For node B :

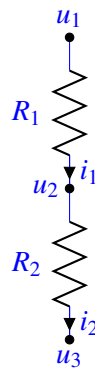
$$0 = \frac{(V_B - V_2)}{R_2} + \frac{(V_B - V_A)}{R_1}$$
$$\frac{1}{R_2}V_2 = V_B \left(\frac{1}{R_2} + \frac{1}{R_1} \right) - V_A \left(\frac{1}{R_1} \right)$$
$$0.5V_2 = -V_A + 1.5V_B$$

6. Flows Through Circuits (12 Points)



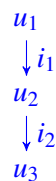
(a) (2 points) Redraw the circuit shown above with all current sources nulled.

Solutions:



(b) (5 points) Translate your circuit from part (a) into a graph and write its incidence matrix \mathbf{F} .

Solutions:



The incidence matrix for the above graph is:

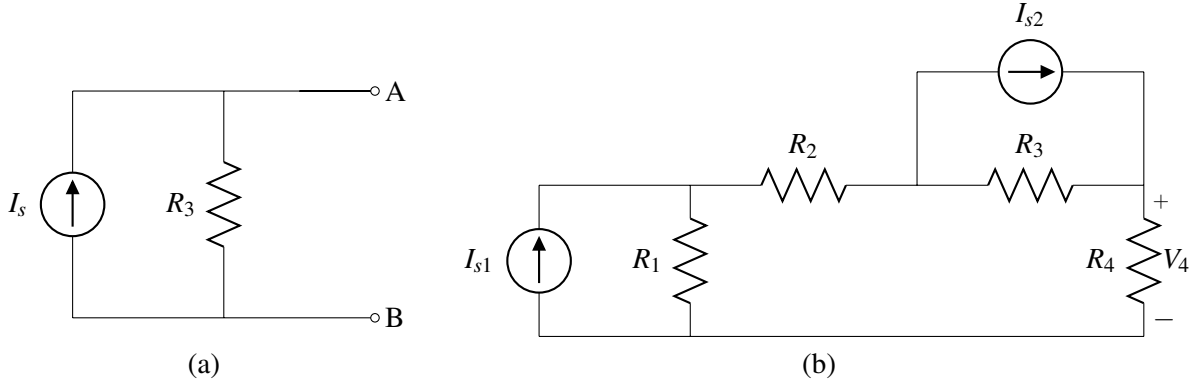
$$\mathbf{F} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

(c) (5 points) Recall that KCL can be stated as $\mathbf{F}^T \vec{i} = 0$ when all the sources are turned off. However, for circuits containing independent current sources, the equation is modified to be $\mathbf{F}^T \vec{i} = \vec{b}$. Find \vec{b} such that $\mathbf{F}^T \vec{i} = \vec{b}$ represents the KCL constraint at every node in the above circuit.

Solutions: I_{s1} flows into node 1 and out of node 3. I_{s2} flows into node 2 and out of node 3. Therefore, the vector \vec{b} is:

$$\vec{b} = \begin{bmatrix} I_{s1} \\ I_{s2} \\ -I_{s1} - I_{s2} \end{bmatrix}$$

7. One Norton At A Time! (12 Points)



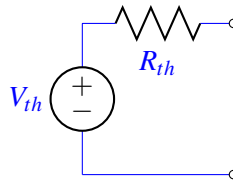
- (a) (2 points) Given the circuit labeled (a), draw the Thevenin equivalent circuit for $I_s = \frac{1}{5} \text{A}$ and $R_3 = 20 \Omega$ and determine V_{th} and R_{th} .

Solutions:

Note that this is a Norton equivalent circuit. We can use $V_{th} = I_{no}R_{no}$ and $R_{th} = R_{no}$ to determine the Thevenin equivalent circuit.

$$V_{th} = V_{AB} = I_s R_3 = 4 \text{ V}$$

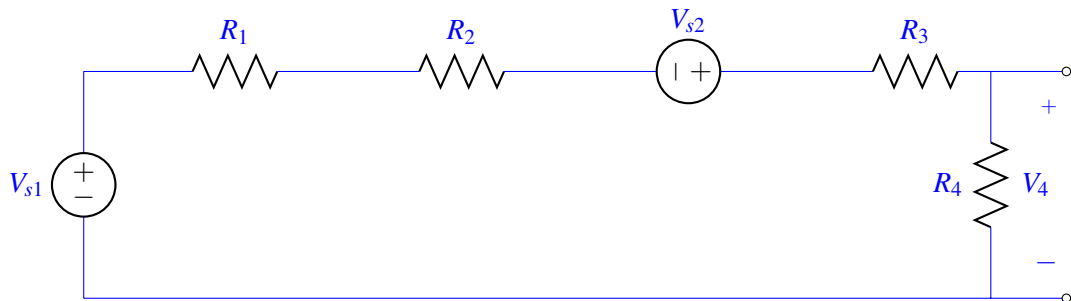
$$R_{th} = R_3 = 20 \Omega$$



- (b) (5 points) Redraw the circuit labeled (b) with the following restrictions: you must use only voltage sources and the given four resistors. Determine the value of the voltage sources given $R_1 = 30 \Omega$, $R_2 = 10 \Omega$, $R_3 = 20 \Omega$, $R_4 = 10 \Omega$, $I_{s1} = \frac{1}{3} \text{A}$, and $I_{s2} = \frac{1}{5} \text{A}$. **Circuits drawn without R_1, R_2, R_3 , and R_4 will not be given full credit. Do not combine resistors.**

Solutions:

I_{s1} and R_1 can be transformed from a Norton equivalent circuit to a Thevenin equivalent circuit. We can do the same for I_{s2} and R_3 .

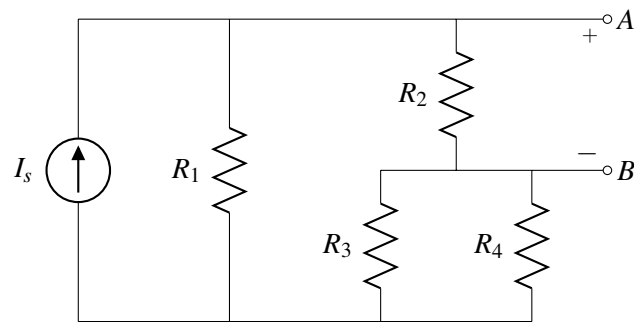


$$V_{s1} = 10\text{ V and } V_{s2} = 4\text{ V}$$

(c) (5 points) Calculate the voltage drop across R_4 .

Solutions: $V_4 = (10\text{ V} + 4\text{ V}) \frac{R_4}{R_1 + R_2 + R_3 + R_4} = \frac{140\text{ V}}{70} = 2\text{ V}$

8. Equivalent Circuits (13 Points)



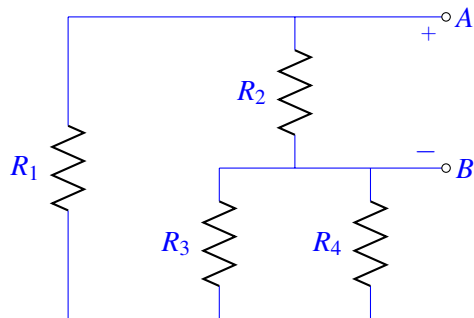
Find V_{th} and R_{th} with respect to nodes A and B for the following circuit in terms of the variables $I_s, R_1, R_2, R_3,$ and R_4 . You may leave the parallel operator \parallel in your final answer.

Solutions: The Thevenin voltage is the voltage across R_2 . To find the V_{R_2} we begin by finding the current through R_2 using the current divider formula.

$$I_{R_2} = \frac{R_1}{R_1 + R_2 + (R_3 \parallel R_4)} I_s$$

$$V_{th} = V_{R_2} = R_2 I_{R_2} = \frac{R_1 R_2}{R_1 + R_2 + (R_3 \parallel R_4)} I_s$$

To find the Thevenin resistance, we begin by removing all independent source. The circuit has been drawn below.



From the above circuit, the Thevenin Resistance $R_{th} = R_2 \parallel (R_1 + (R_3 \parallel R_4))$

9. Resist the Touch (16 Points)

In this question, we will be re-examining the 2-dimensional resistive touchscreen previously discussed in both lecture and lab. The general touch screen is shown in Figure 1 (a). The touchscreen has length L and width W and is composed of a rigid bottom layer and a flexible upper layer. The strips of a single layer are all connected by an ideal conducting plate on each side. The upper left corner is position $(1, 1)$.

The top layer has N vertical strips of denoted by y_1, y_2, \dots, y_N . These vertical strips have cross sectional area A , and resistivity ρ_y .

The bottom layer has N horizontal strips denoted by x_1, x_2, \dots, x_N . These horizontal strips all have cross sectional area A as well, and resistivity ρ_x .

Assume that all top layer resistive strips and bottom layer resistive strips are spaced apart equally. Also assume that all resistive strips are rectangular as shown by Figure 1 (b).

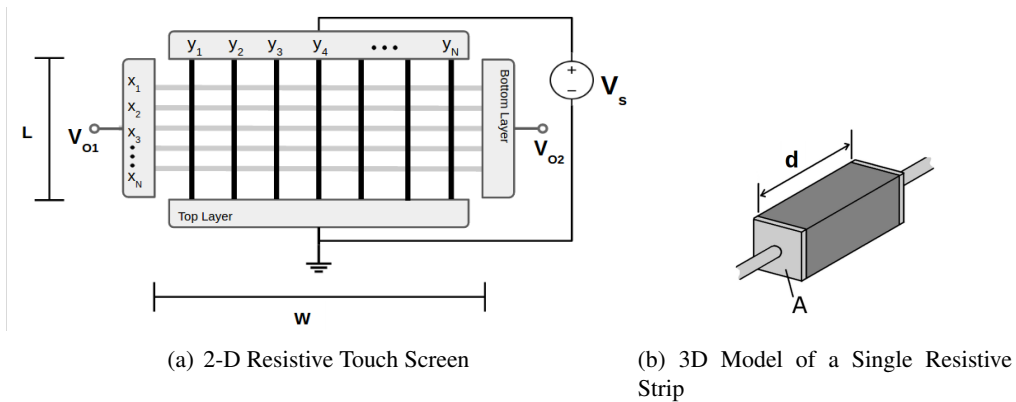


Figure 1:

- (a) (3 points) Figure 1(b) shows a model for a single resistive strip. Find the equivalent resistance R_y for the vertical strips and R_x for the horizontal strips, as a function of the screen dimensions W and L , the respective resistivities, and the cross-sectional area A .

Solutions: The equation for resistance is $R = \frac{\rho l}{A}$

Therefore, $R_y = \frac{\rho_y L}{A}$.

For the bottom, $R_x = \frac{\rho_x W}{A}$.

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(b) (5 points) Consider a 2×2 example for the touchscreen circuit.

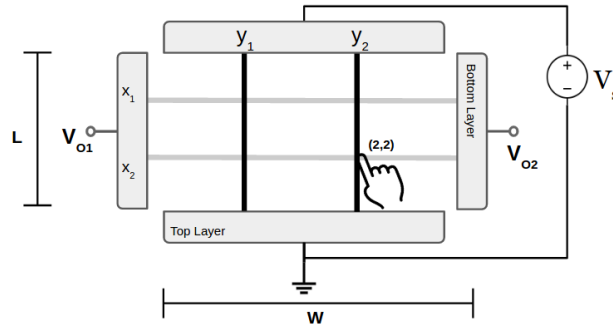


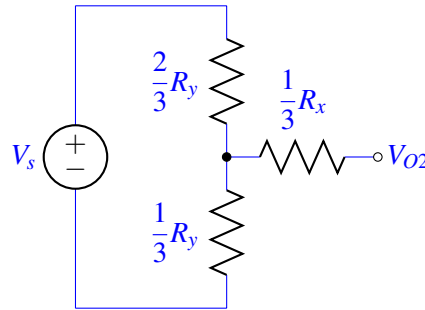
Figure 2: 2×2 Case of the Resistive Touchscreen

Given that $V_s = 3\text{V}$, $R_x = 2000\Omega$, and $R_y = 2000\Omega$, draw the equivalent circuit for when the point (2,2) is pressed and solve for the voltage at terminal V_{O2} with respect to ground.

Solutions:

Since all of the resistive strips are equally spaced, the resistor above point (2,2) on strip y_2 becomes $\frac{2}{3}R_y$ and the resistor below point (2,2) on strip y_2 becomes $\frac{1}{3}R_y$.

The bottom layer resistors, although they must be drawn in the equivalent circuit, do not affect the voltage at V_{O1} as they are open circuits.



Observing that the resistive strips form a voltage divider, we can determine V_{O2} using the voltage divider equation.

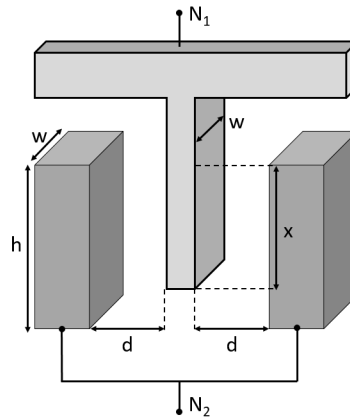
$$\text{Therefore, } V_{O2} = V_{(2,2)} = V_s \frac{\frac{1}{3}R_y}{\frac{1}{3}R_y + \frac{2}{3}R_y} = \frac{1}{3}V_s = 1\text{V}.$$

(c) (8 points) Suppose a touch occurs at coordinates (i, j) in Figure 1(a). Find an expression for V_{O2} as a function of V_s , N , i , and j .

Solutions:

$$\begin{aligned} V_{O2} &= \frac{N+1-i}{N+1} R_y V_s \\ &= \frac{N+1-i}{N+1} V_s \end{aligned}$$

10. MEMS Sensor (20 Points)



Above is the cross section of a MEMS accelerometer, which consists of a T-shaped mass between two sidewalls. When the device accelerates, the T structure, which is held up by springs, displaces up or down. Assume that all the structures are made of metal separated by air, and that the only relevant capacitances in this problem are parallel plate capacitances.

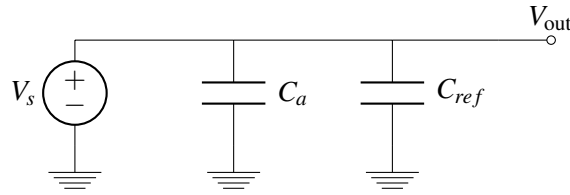
- (a) (4 points) Suppose both sidewalls are connected together with an ideal wire. Find the capacitance between the stem of the T mass and a sidewall as a function of x , the displacement of the T mass. Find the net capacitance between the two terminals N_1 and N_2 assuming the only parallel plate capacitance is that of the T mass stem and sidewall.

Solutions: The capacitance between the T and a sidewall is found using the equation for physical capacitance.

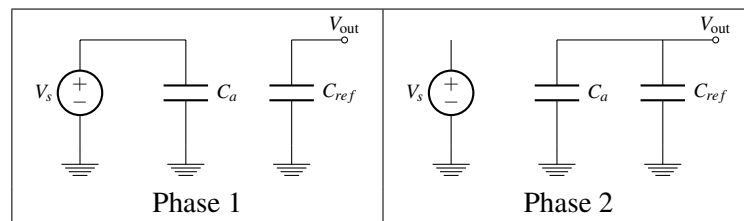
$$C = \epsilon_0 \frac{Wx}{d}$$

There are two such capacitors in the above structure, and the net capacitance is $2C$.

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The above circuit is designed to measure the displacement of the T structure. It cycles through two phases, which are depicted in the diagram below. For this problem, you may assume that switching is controlled through a microcontroller and that all capacitors reach steady state during each phase. Furthermore, assume that the T mass does not move during the measurement cycle and all capacitors are initially uncharged.



- (b) (8 points) The circuit above is designed to measure the change in capacitance and to output a voltage. The phases of its operation are shown above. Find V_{out} in Phase 2 as a function of C_a and C_{ref} .

Solutions: In Phase 1, the charge on C_{ref} is not connected to a voltage source, so it has no additional charge on it. Since initially all capacitors were uncharged, the charge on C_{ref} is 0. C_a is connected to a voltage source, and has charge $C_a V_s$ on it. We use Q_{tot} to denote the total charge on the nodes that will be shared.

$$Q_{tot,1} = Q_a + Q_{ref} = C_a V_s + 0$$

In Phase 2, the two capacitor charge share. Since they are now connected in parallel, they have the same voltage, V_{out} across them.

$$Q_{tot,2} = Q_a + Q_{ref} = (C_a + C_{ref}) V_{out}$$

$$Q_{tot,2} = Q_{tot,1}$$

$$(C_a + C_{ref}) V_{out} = C_a V_s$$

$$V_{out} = \frac{C_a}{C_a + C_{ref}} V_s$$

- (c) (8 points) Let $V_{out}[i]$ denote V_{out} at the i th measurement cycle. Suppose the circuit had been running for k cycles. What is $V_{out}[k]$ in terms of $V_{out}[k-1]$, C_a and C_{ref} ?

Solutions: Here, unlike before there is some charge on C_{ref} because C_{ref} is not being discharged.

The charge on C_{ref} in Phase 1 can be found using the voltage at cycle $k-1$.

$$Q_{tot,1} = C_a V_s + C_{ref} V_{out}[k-1]$$

$$Q_{tot,2} = C_a V_{out}[k] + C_{ref} V_{out}[k]$$

$$Q_{tot,2} = Q_{tot,1}$$

$$C_a V_{out}[k] + C_{ref} V_{out}[k] = C_a V_s + C_{ref} V_{out}[k-1]$$

$$V_{out}[k] = \frac{C_a}{C_a + C_{ref}} V_s + \frac{C_{ref}}{C_a + C_{ref}} V_{out}[k-1]$$