Midterm 2 Solution

PRINT your student ID: ____

PRINT AND SIGN your name: ____________________, ____________________ ______________________
(last name) (first name) (signature)

PRINT your discussion section and GSI(s) (the one you attend): ____

Name and SID of the person to your left: ____

Name and SID of the person to your right: ____

Name and SID of the person in front of you: ____

Name and SID of the person behind you: ____

1. What do you enjoy most about EE16A? (1 Point)

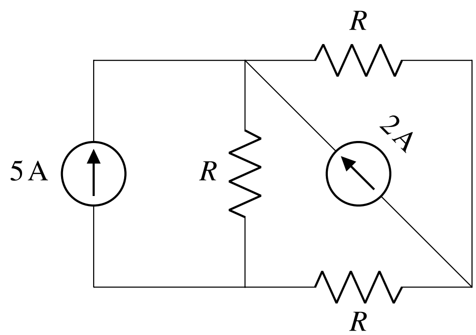
2. What other courses are you taking this semester? (1 Point)

Do not turn this page until the proctor tells you to do so. You may work on the questions above.

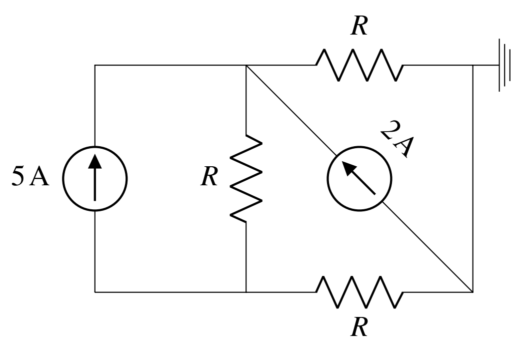
3. Nodal Analysis (6 Points)

Your friends, Anant and Elad, are attempting to solve the circuit below using the nodal analysis technique you learned in lecture. However, they got stuck on some steps and need your help!

In the following parts, they want to know whether their work is correct or not. For each part, *circle the correct answer and include a brief justification (fewer than 20 words) explaining your choice.*



(a) (2 Points) Elad first grounds the circuit, such that it looks like the one below. Anant, who is used to circuit diagrams with the ground at the bottom of the circuit, wonders if we can put the ground off to the side.



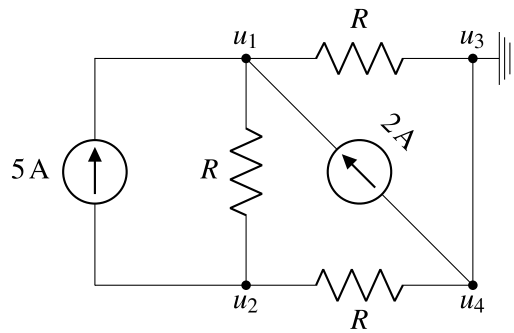
Did Elad choose to label the ground node at a valid location?

YES **NO**

Solution:

Yes, we can place the ground anywhere in the circuit.

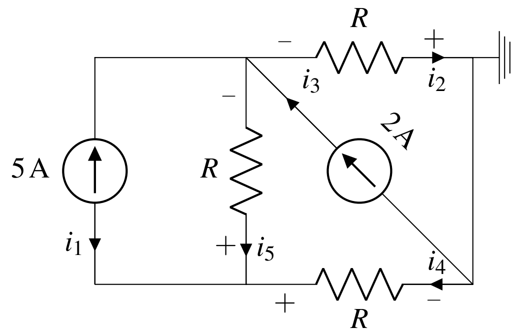
(b) (2 Points) Anant then adds four labels u_1 through u_4. Are any of these labels redundant (i.e., are any of the nodes in the circuit labeled more than once)?



YES **NO**

Solution:
Yes, u_3, u_4 and ground are all the same node.

(c) (2 Points) Elad then labels the currents, i_1 through i_5, and adds the $+/-$ signs for the resistors while attempting to obey passive sign convention. Did he follow passive sign convention correctly for all of the resistors?



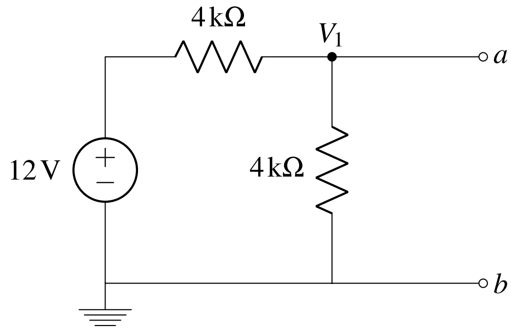
YES **NO**

Solution:
No, he is incorrect. Passive sign convention dictates that the current should enter the positive terminal and exit the negative terminal, but in his labeling, it enters the negative terminal and exits the positive terminal.

PRINT your name and student ID: ____

4. Thévenin and Norton Circuits (13 Points)

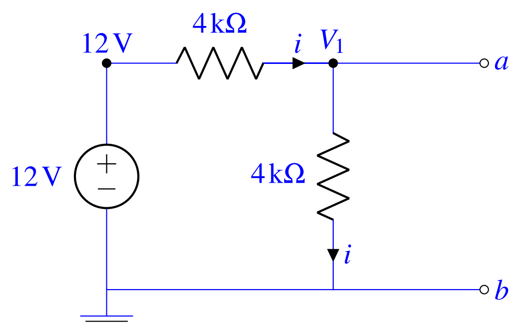
Consider the following circuit:



(a) (3 Points) *Find the voltage* V_1 (relative to ground).

Solution:

This circuit can be solved using nodal analysis. Notice that with ground and V_1 labeled, the only other node is on top of the voltage source. Since this node is directly on top of a voltage source, we do not need to label it. Since all junctions in this circuit are "trivial," we do not need to write any KCL equations either. We will label the current i and label the voltages across the resistors as follows:



From the I-V relationships for resistors, we find the following equations:

$$\begin{cases} 12 - V_1 = 4k \cdot i \\ V_1 - 0 = 4k \cdot i \end{cases}$$

Since we have two equations with two unknowns, we can solve for the voltage V_1.

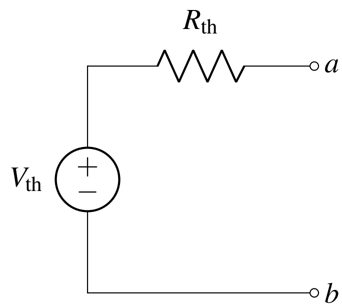
$$12 - 2V_1 = 0$$

$$V_1 = 6\,\text{V}$$

You can also observe that this circuit is a voltage divider. Therefore,

$$V_1 = \frac{4\,\text{k}\Omega}{4\,\text{k}\Omega + 4\,\text{k}\Omega} \cdot 12\,\text{V} = 6\,\text{V}$$

(b) (4 Points) *Calculate* R_{th} *and* V_{th} such that the Thévenin equivalent circuit shown below matches the I-V characteristics of the original circuit between the a and b terminals.

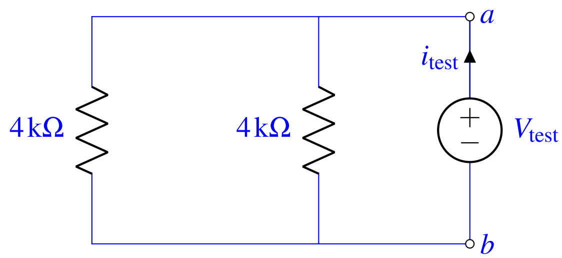


Solution:

First, we solve for V_{th}, which is equal to V_{oc}.

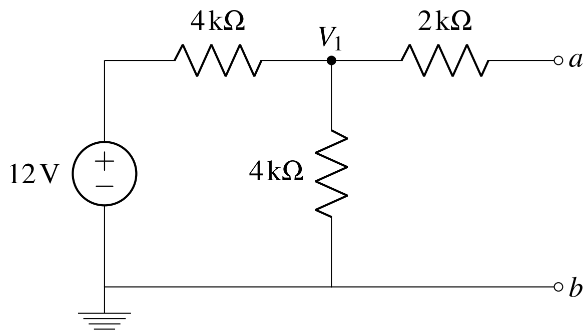
$$V_{\text{th}} = V_{oc} = V_1 = 6\,\text{V}$$

Next, we solve for R_{th}. We first redraw the circuit after nulling all independent sources and adding a test voltage source, V_{test}.

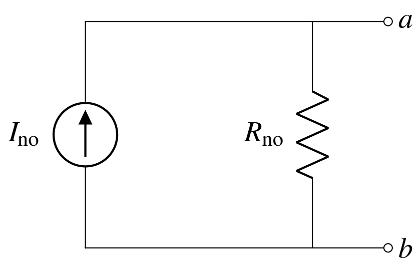


Notice the votlage at the node at the top is V_{test}. The current through each $4\,\text{k}\Omega$ resistor is $\frac{V_{\text{test}}}{4\,\text{k}\Omega}$ The current i_{test} out of the voltages source is then equal to the sum of the two currents or $\frac{V_{\text{test}}}{2\,\Omega}$. Therefore, the Thévenin equivalent resistance is $R_{\text{th}} = \frac{V_{\text{test}}}{i_{\text{test}}} = 2\,\Omega$.

(c) (6 Points) As shown below, we will now consider what happens when we add another resistor to the original circuit.



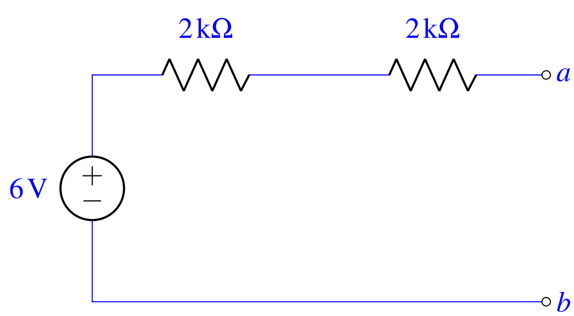
Find the values of R_{no} *and* I_{no} such that the Norton equivalent circuit shown below matches the *I*-*V* characteristics of this new circuit between the *a* and *b* terminals.



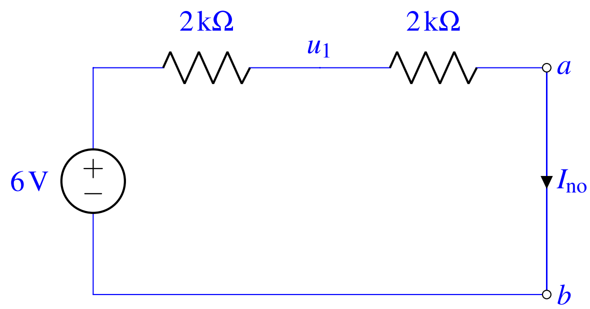
Hint: Your result from part (b) might be useful.

Solution:

First, we solve for I_{no}, which is equal to I_{sc}. Since we calculated the Thévenin equivalent circuit in the previous part, we can represent this circuit as follows.



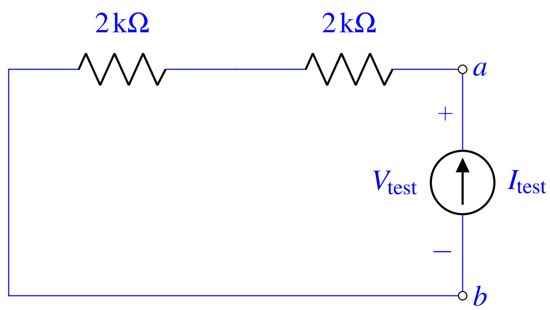
Shorting the nodes *a* and *b*, we find the circuit shown below.



Applying the nodal analysis procedure, using KCL at u_1 and ohm's law we find $\frac{6-u_1}{2\,\text{k}\Omega} = \frac{u_1}{2\,\text{k}\Omega}$.
From the above equation, we know u_1 is $3V$, which tells us the current through the bottom resistor.

$$I_R = I_{sc} = I_{\text{no}} = \frac{3\,\text{V}}{2\,\text{k}\Omega} = 1.5\,\text{mA}$$

To find R_{no}, we begin by turning off the voltage source and applying a test current source to the nodes a and b.



Each resistor above has I_{test} flowing through it, the the votlage across each resistor is $2\,\text{k}\Omega I_{\text{test}}$. The overall V_{test} is then the sum $V_{\text{test}} = 2\,\text{k}\Omega I_{\text{test}} + 2\,\text{k}\Omega I_{\text{test}}$

$$R_{\text{no}} = \frac{V_{\text{test}}}{I_{\text{test}}} = 2\,\text{k}\Omega + 2\,\text{k}\Omega = 4\,\text{k}\Omega$$

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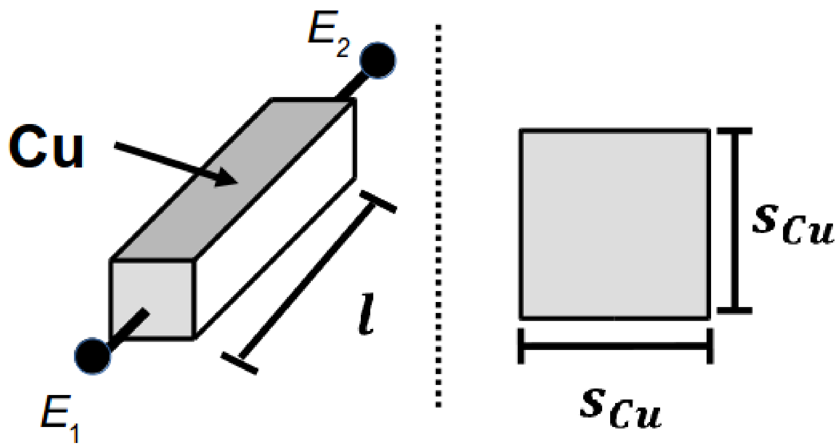
5. Wire we doing this... (13 Points)

A common structure used in the field of nanotechnology research is something called a core-shell nanowire. This consists of a physical structure that has a core made of one material and a shell made of another, where current flows through both parts. Note that the following figures are not drawn to scale.

(a) (3 Points) A copper (Cu) structure with a square cross-section is shown below. Given the material parameters, *calculate the resistance* R_{Cu} of the structure between E_1 and E_2.

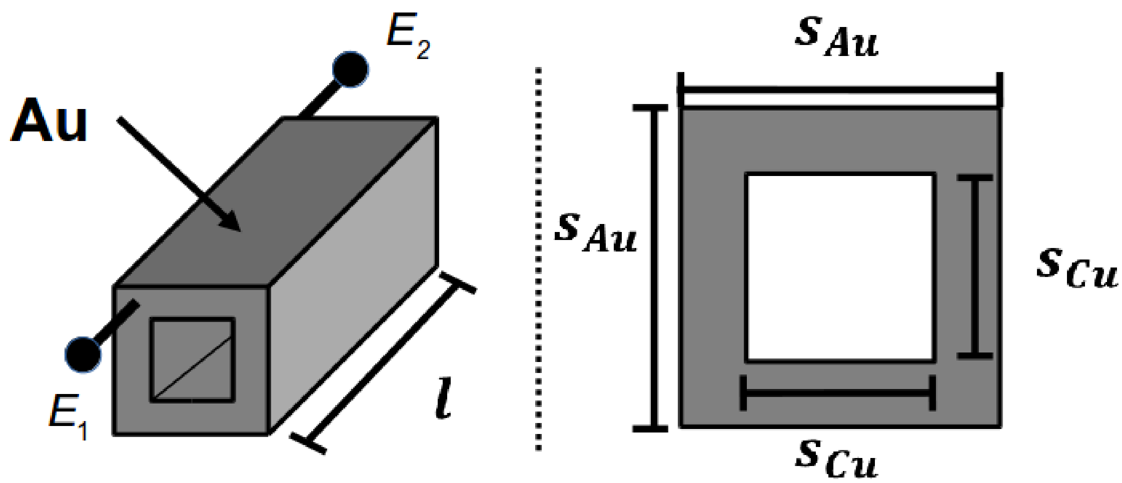


ρ_{Cu}	$1\times10^{-8}\,\Omega\,\text{m}$
s_{Cu}	$5\,\text{nm}$
l	$75\,\text{nm}$

Solution:
Using the formula for resistance R, we can find the resistance of the Cu structure given the resistivity and the dimensions:

$$R=\rho\frac{l}{A} \implies R_{\text{Cu}}=\rho_{\text{Cu}}\frac{l}{s_{\text{Cu}}^2}=1\times10^{-8}\,\Omega\,\text{m}\cdot\frac{75\times10^{-9}\,\text{m}}{\left(5\times10^{-9}\,\text{m}\right)^2}=30\,\Omega$$

(b) (4 Points) A gold (Au) structure in the shape of a shell is shown below. Given the material parameters, *calculate the resistance* R_{Au} of the Au structure between E_1 and E_2.



ρ_{Au}	$2 \times 10^{-8}\,\Omega\,\text{m}$
s_{Au}	$10\,\text{nm}$
l	$75\,\text{nm}$

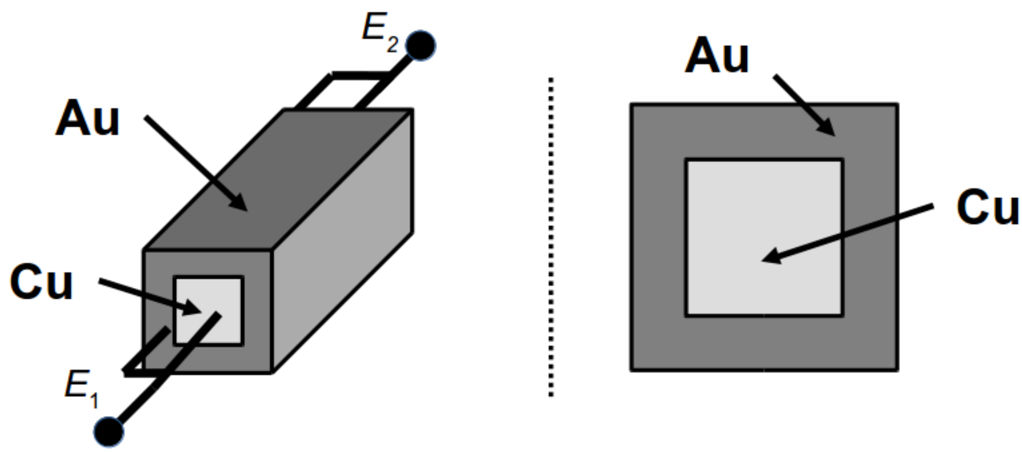
Solution:

Using the formula for resistance R, we can find the resistance of the Au structure given the resistivity and the dimensions. However, in this case, the area of the Au structure is not just a square but rather the area of the Cu square subtracted from the area of the Au square (the area of a shell).

$$R = \rho \frac{l}{A}$$

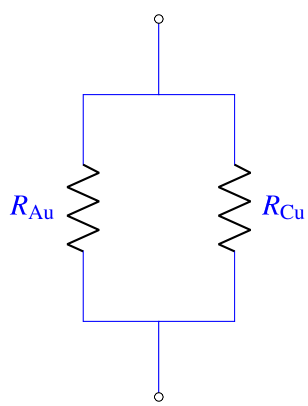
$$R_{\text{Au}} = \rho_{\text{Au}} \frac{l}{s_{\text{Au}}^2 - s_{\text{Cu}}^2} = 2 \times 10^{-8}\,\Omega\,\text{m} \cdot \frac{75 \times 10^{-9}\,\text{m}}{\left(10 \times 10^{-9}\,\text{m}\right)^2 - \left(5 \times 10^{-9}\,\text{m}\right)^2} = 20\,\Omega$$

(c) (3 Points) Now the two structures are combined together, such that they make one structure, with the outside shell made of Au and the inside made of Cu. This is called a core-shell nanowire. Assuming that you are contacting the full ends of the nanowire (i.e., E_1 and E_2 are both connected with ideal wires to the faces of the Cu and Au structure), *model the nanowire as a set of resistors*, using R_{Au} for the resistance of the Au layer and R_{Cu} for the resistance of the Cu layer.



Solution:

Given that we are contacting the full area and that current is flowing from end to end, each end can be treated as a node since each end will have the Au and Cu at the same potential. This means that we can model the core-shell nanowire as a set of parallel resistors.



(d) (3 Points) Based on your model from part (c), find the equivalent resistance R_{wire} between E_1 and E_2.

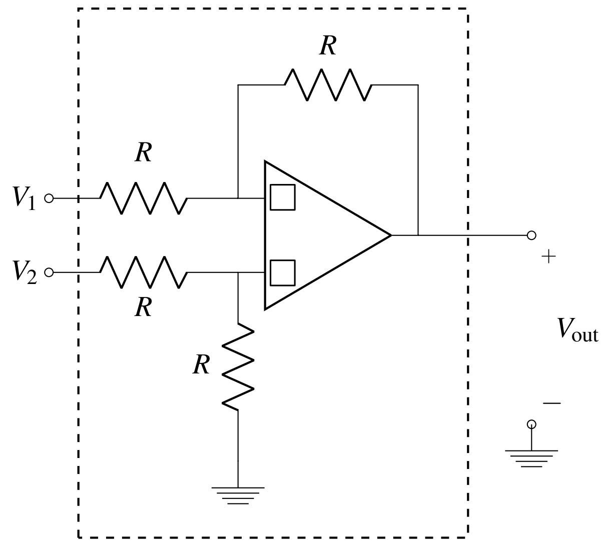
Solution:

Because the two structures are in parallel, the total resistance R_{wire} is:

$$R_{\text{wire}} = R_{\text{Au}} \parallel R_{\text{Cu}} = \frac{20\,\Omega \cdot 30\,\Omega}{20\,\Omega + 30\,\Omega} = \frac{600\,\Omega^2}{50\,\Omega} = 12\,\Omega$$

6. Do You See The Difference? (11 Points)

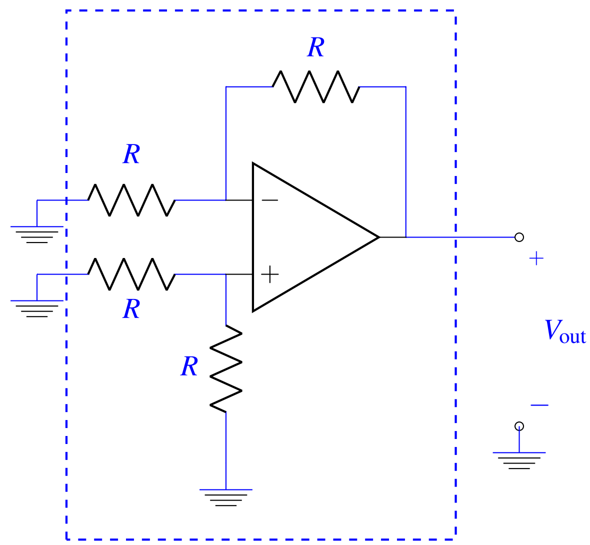
Consider the following circuit:



(a) (4 Points) *Label the '+' and '−' terminals of the op-amp above* so that it is in negative feedback.

Solution:

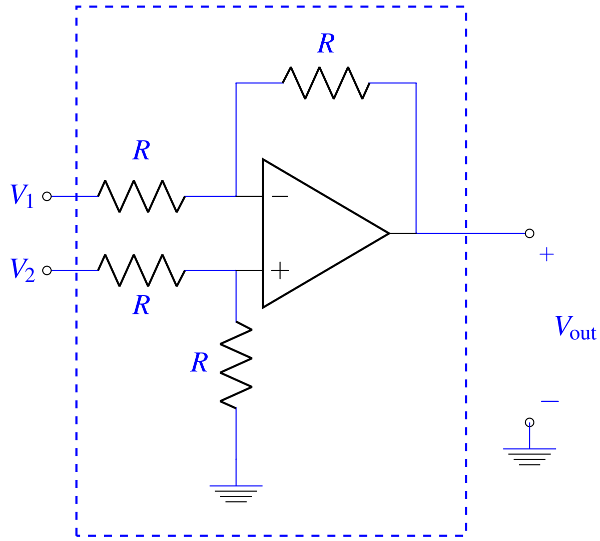
In order for the amplifier to be in negative feedback, we first null all input sources.



We then dink the output V_{out}, so assume that we increase V_{out}. If the amplifier is in negative feedback, we need the output of the amplifier to decrease.

We notice that the two resistors connected to the top input terminal of the op-amp form a voltage divider of V_{out}. If we increase V_{out}, then the voltage at the top input terminal will increase as well.

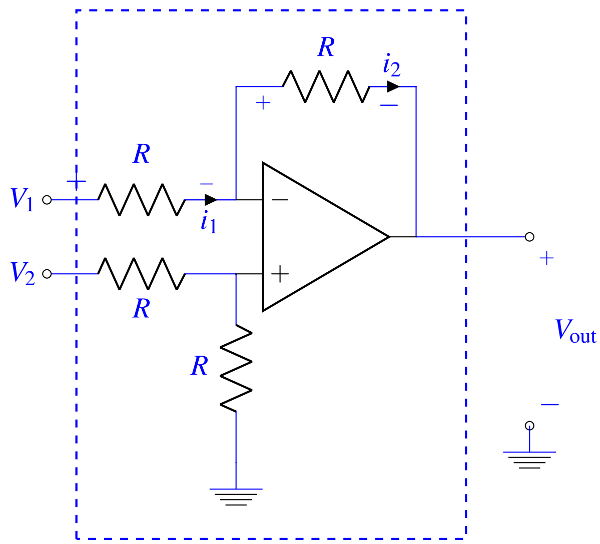
We know that the amplifier amplifies $V^+ - V^-$, so if we want the output of the amplifier to decrease when the voltage at the top input terminal increases, we need the top input terminal to be the negative input terminal. Consequently, the bottom input terminal will be the positive input terminal.



(b) (7 Points) Assuming that the op-amp is in negative feedback, *use the Golden Rules (combined with any other analysis technique) to find* V_{out} in terms of R, V_1, and V_2.

Solution:

Using the Golden Rules and nodal analysis, we can solve for V_{out}.



First, we notice the voltage divider at the positive input terminal of the amplifier. Since there is no current flowing into the input terminals of the op-amp according to the Golden Rules,

$$V^+ = \frac{R}{R+R}V_2 = \frac{V_2}{2}.$$

Now, we can apply KCL and Ohm's law at the negative input terminal. By the Golden Rules, we know that there is no current flowing into the op-amp. Therefore,

$$i_1 = i_2$$

$$\frac{V_1 - V^-}{R} = \frac{V^- - V_{out}}{R}$$

$$V_1 - V^- = V^- - V_{\text{out}}$$

We know that the op-amp is in negative feedback, so by the Golden Rules, $V^- = V^+ = \frac{V_2}{2}$.

$$V_1 - \frac{V_2}{2} = \frac{V_2}{2} - V_{\text{out}}$$

$$V_1 = V_2 - V_{\text{out}}$$

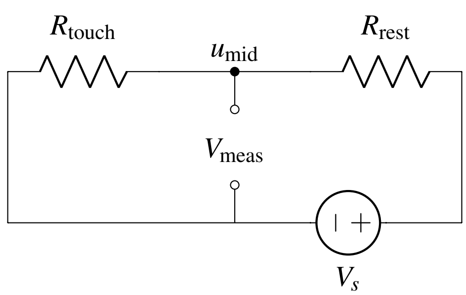
$$V_{\text{out}} = V_2 - V_1$$

7. A New Feature You Didn't Even Know You Wanted! (14 Points)

An up-and-coming computer company, Orange Inc., is trying to design a touchscreen bar to incorporate into their new laptop, right above the keyboard. Let's help them analyze their existing design to see where their design has gone wrong!

(a) (7 Points) Orange Inc.'s touchscreen is small enough that we are only interested in the horizontal position of the touch and hence can use the 1D touchscreen circuit model shown below, where the u_{mid} node is labeled at the point the touch occurs. The touchscreen bar has a total length of 10 cm, but due to some disputes with their supplier, Orange Inc. has not been able to find out what the resistivity of the touchscreen material is. Despite this, your colleague claims that they can still predict the relationship between V_{meas} and the position where a customer touched the bar. Is your colleague correct? *Circle your answer.*

If you answered that your colleague is correct, *provide an expression for V_{meas} as a function of V_s and the position of the touch x* (measured in cm relative to the *left* side of the circuit). If you answered that your colleague is incorrect, *provide an expression for V_{meas}* as a function of V_s, R_{touch}, and R_{rest}.



Solution:

Yes, we can determine the position where the customer touched the bar by just measuring V_{meas}.

$$R_{\text{touch}} = \rho(\Omega \times cm)\frac{x(cm)}{A(cm^2)}$$

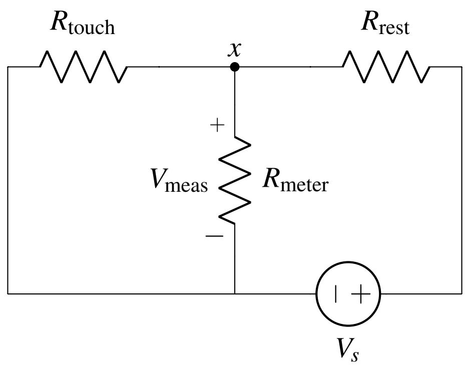
$$R_{\text{rest}} = \rho(\Omega \times cm)\frac{(10(cm) - x(cm))}{A(cm^2)}$$

Recognizing that this circuit is a voltage divider, we find

$$V_{\text{meas}} = \frac{R_{\text{touch}}}{R_{\text{touch}} + R_{\text{rest}}}V_s = \frac{\frac{\rho x}{A}}{\frac{\rho x}{A} + \frac{\rho(10-x)}{A}}V_s = \frac{x}{10}V_s$$

We know this answer makes sense since x and 10 are in cm, so $\frac{x}{10}V_s$ has units of volts.

(b) (7 Points) It turns out that Orange Inc's problems aren't limited to their touchscreen materials – the device they use to measure the voltage V_{meas} has a finite but known resistance R_{meter} associated with it. Connecting the measurement device to the touchscreen results in the circuit model shown below. Without knowing the value of the resistivity of the material (which, as a reminder, would affect the values of R_{touch} and R_{rest}), can you compute the value of V_{meas}? *Justify your answer by providing an expression for* V_{meas} as a function of R_{touch}, R_{rest}, R_{meter}, and V_s.



Solution:

No, we can no longer determine V_{meas}.

$$V_{\text{meas}} = \frac{R_{\text{touch}} \parallel R_{\text{meter}}}{R_{\text{touch}} \parallel R_{\text{meter}} + R_{\text{rest}}} V_s = \frac{\frac{R_{\text{touch}} R_{\text{meter}}}{R_{\text{touch}} + R_{\text{meter}}}}{\frac{R_{\text{touch}} R_{\text{meter}}}{R_{\text{touch}} + R_{\text{meter}}} + R_{\text{rest}}} V_s = \frac{R_{\text{touch}} R_{\text{meter}}}{R_{\text{touch}} R_{\text{meter}} + R_{\text{touch}} R_{\text{rest}} + R_{\text{meter}} R_{\text{rest}}} V_s$$

We can no longer determine the position where the customer touched the bar because ρ and the A will not cancel out in this equation.

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8. Force Touch (22 Points)

So far, our capacitive touchscreens have been able to measure the presence or absence of a touch, but with some modifications, we can actually measure how hard the finger is pressing (i.e., force) as well. Figure 8.1 shows this type of touch screen without any touch and with the finger pressing on it; the more force the finger applies to the screen, the more the distance between the two metal plates decreases.

Assume that the insulator in between the plates has some permittivity ε_1 and that the top metal plate has an area A. With no force applied on the screen, the top and bottom plates are a distance d apart. When a force is applied, the distance becomes d' ($< d$). Suppose when a finger is touching the screen, it creates a capacitance $C_{F,E_{\text{top}}}$ between itself and the top plate and a capacitance $C_{F,E_{\text{bottom}}}$ between itself and the lower plate.

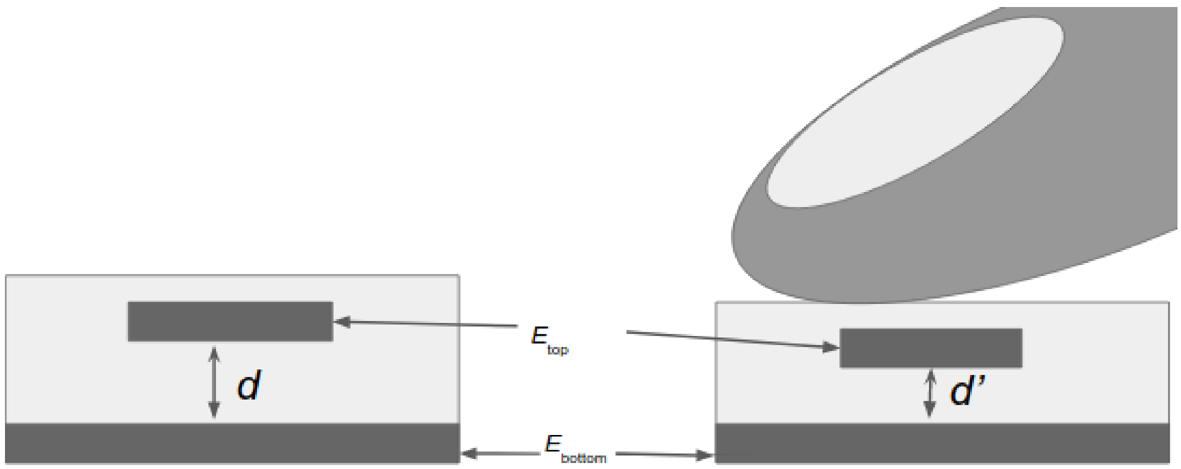


Figure 8.1: Sensor configurations.

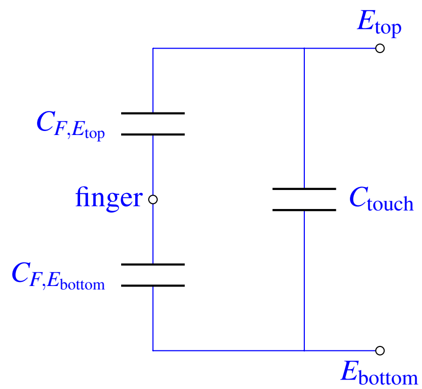
(a) (3 Points) With no finger touching or applying any force, *find the capacitance* $C_{no\ touch}$ between the top metal plate and the bottom metal plate. Express your answer in terms of ε_1, d, and A.

Solution:

$$C_{\text{no touch}} = \frac{\varepsilon_1 A}{d}$$

(b) (4 Points) Now suppose that a finger that is touching the screen applies some force on our screen. *Draw a circuit model* including all of the capacitors connected to either E_{top} or E_{bottom}. *Label all elements* in your model.

Solution:

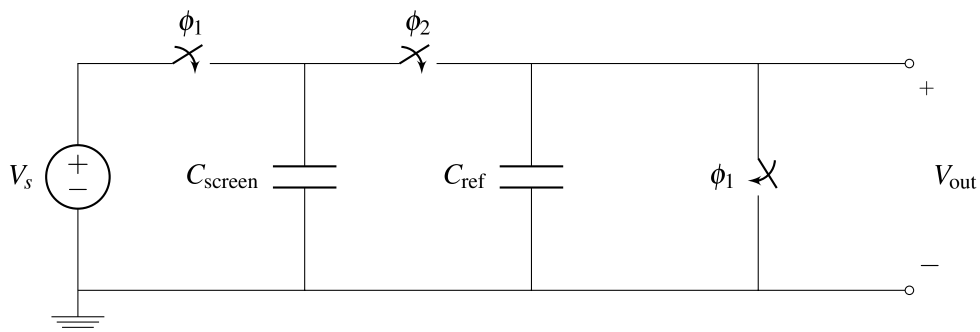


(c) (4 Points) Assuming that $C_{F,E_{\text{top}}} = C_{F,E_{\text{bottom}}} = 0\,\text{F}$, *find the equivalent capacitance, C_{force},* between E_{top} and E_{bottom}. Express your answer in terms of ε_1, d', and A.

Solution:

$$C_{\text{force}} = C_{\text{touch}} = \frac{\varepsilon_1 A}{d'}$$

(d) (4 Points) We connect our structure to the circuit shown below, where your answer to part (c) is now some C_{screen} (which represents the equivalent capacitance between E_{top} and E_{bottom}). The circuit cycles through two phases. In phase 1, switches labeled ϕ_1 are **on**, and in phase 2, switches labeled ϕ_2 are **on**. *Derive the value of V_{out} during phase 2* in terms of C_{screen}, C_{ref} and V_s.



Solution:

In phase 1, $Q_{\text{screen}} = C_{\text{screen}} V_s$ and $Q_{\text{ref}} = 0$, so $Q_{\text{tot},1} = C_{\text{screen}} V_s$.

In phase 2, $V_{\text{Cscreen}} = V_{\text{Cref}} = V_{\text{out}}$, so $Q_{\text{tot},2} = (C_{\text{screen}} + C_{\text{ref}}) V_{\text{out}}$.

By conservation of charge,

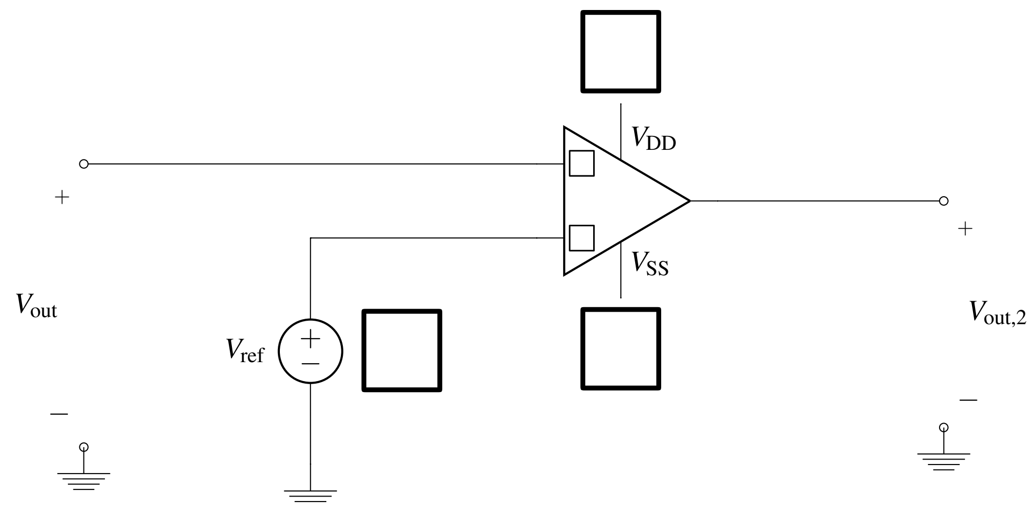
$$Q_{\text{tot},1} = Q_{\text{tot},2}$$

$$C_{\text{screen}} V_s = (C_{\text{screen}} + C_{\text{ref}}) V_{\text{out}}$$

$$V_{\text{out}} = \frac{C_{\text{screen}}}{C_{\text{screen}} + C_{\text{ref}}} V_s$$

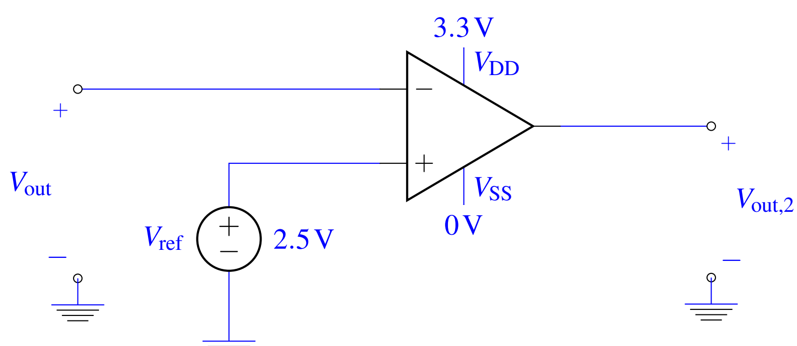
(e) (7 Points) In the previous circuit, if the finger is pressing with a certain force F' and $V_s = 5\,\text{V}$, assume that $V_{\text{out}} = 2.5\,\text{V}$ during phase 2. We want to design a circuit that outputs $0\,\text{V}$ when we apply more force than F' and $3.3\,\text{V}$ when we apply less force than F'.

In the circuit below, *label the terminals of the op-amp, indicate what you will connect its supplies to, and pick a value for V_{ref}* such that $V_{\text{out},2} = 0\,\text{V}$ when more force than F' is applied and $V_{\text{out},2} = 3.3\,\text{V}$ when less force than F' is applied.



Solution:

If more force than F' is applied, d' decreases, so C_{screen} increases, and $V_{\text{out}} > 2.5\,\text{V}$. Since we want to output $0\,\text{V}$ when we apply more force, we need the input terminal connected to the V_{out} to be labeled '−'. We want to output $3.3V$ and $0V$ so VDD will be connected to $3.3V$ and VSS to $0V$.



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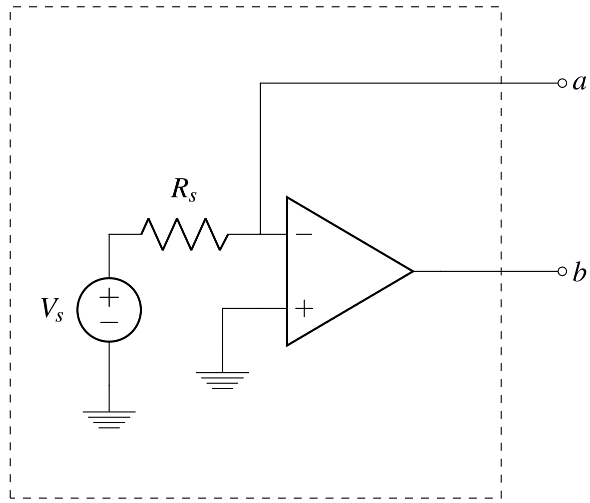
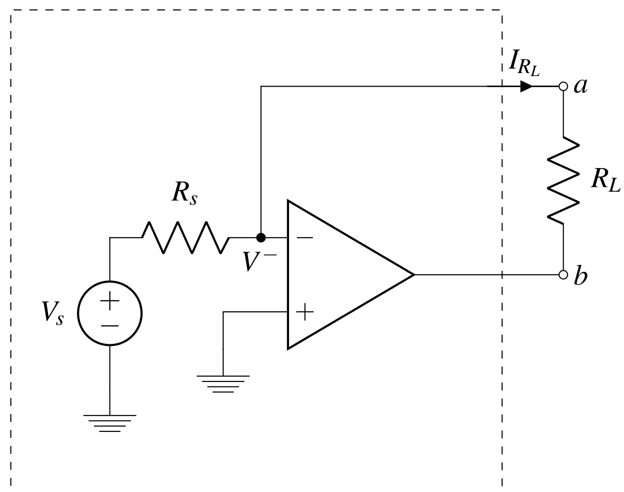
9. Op-Amp Fun (11 Points + 5 Points)

Consider the following circuit:



(a) (3 Points) Suppose that we connect a resistor across the terminals a and b as shown in the circuit below. *Find the voltage V^-* at the inverting input terminal of the op-amp relative to ground.

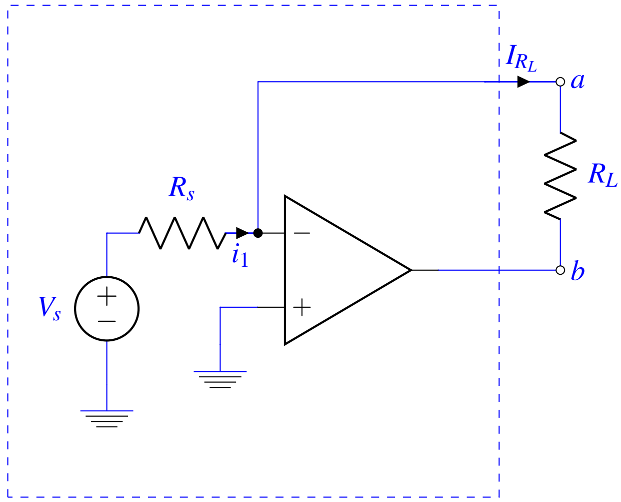


Solution:

Since the op-amp is in negative feedback, we apply the Golden Rules.

$$V^- = V^+ = 0\,\text{V}$$

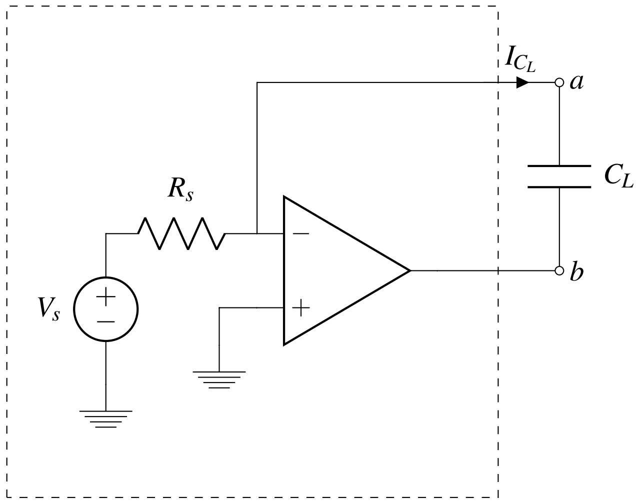
(b) (3 Points) *Find the current* I_{R_L} through the resistor R_L as a function of V_s, R_s, and R_L.
Solution:



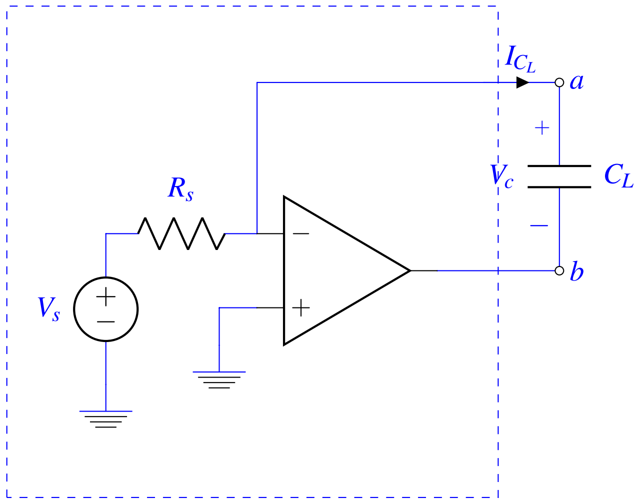
Applying KCL and ohm's law at the inverting input terminal,

$$I_{R_L} = i_1 = \frac{V_s - 0\,\mathrm{V}}{R_s} = \frac{V_s}{R_s}$$

(c) (5 Points) Suppose that you replace R_L with a capacitor C_L as shown below. *Find the current* I_{C_L} flowing through the capacitor C_L as a function of V_s, R_s, and C_L.



Solution:



Even with the capacitor, this circuit remains in negative feedback. We can check this by applying a small increase in V_b. This change in voltage will cause a current to flow through C_L, opposite to the direction of the labeled I_{C_L}. This current will then flow through R_s increasing the voltage across R_s, and increasing V^-. Since the amplifier amplifies $V^+ - V^-$, the output will decrease. Thus the circuit is in negative feedback.

Since the current out the capacitor is labeled in the same direction, we can apply the same procedure we did earlier. Applying KCL and Ohm's law at the inverting input terminal,

$$I_{C_L} = \frac{V_s}{R_s}$$

(d) (Bonus: 5 Points) Assume that at time $t = 0$, the voltage across C_L is equal to $0\,\text{V}$. *Derive an expression for the voltage V_b at node b* (relative to ground) as a function of V_s, R_s, C_L, and t.

Solution:

$$I_{C_L} = \frac{V_s}{R_s} = C_L \frac{dV_c}{dt} = C_L \frac{d(V^- - V_b)}{dt} = -C_L \frac{dV_b}{dt}$$

$$\frac{dV_b}{dt} = -\frac{V_s}{R_s C_L}$$

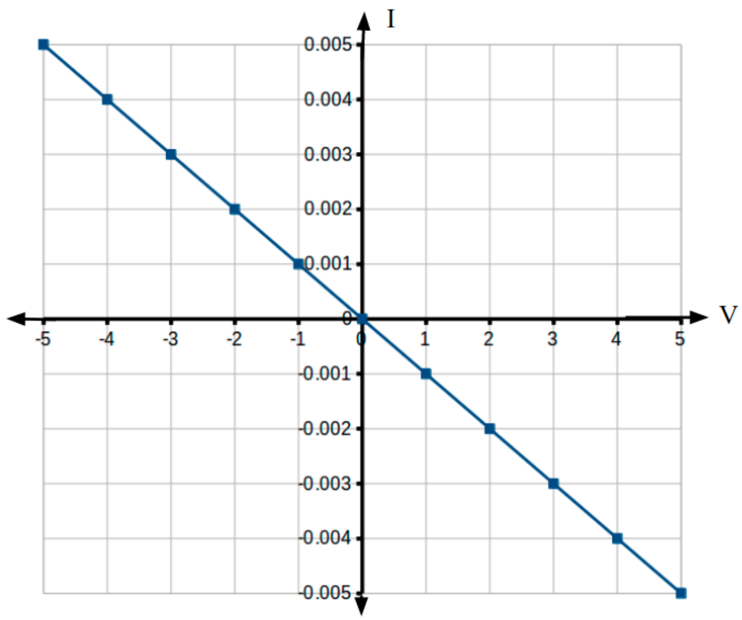
$$V_b = \int_0^t -\frac{V_s}{R_s C_L} = -\frac{V_s t}{R_s C_L}$$

PRINT your name and student ID: ____

10. Are You Resistive? (19 Points + 5 Points)

Bob is a quality control engineer, and his job is to document and analyze the test results of resistors made by his company.

(a) (8 Points) One day, Bob was testing the I-V curves of the resistors, and he saw something surprising for one particular resistor R_{special}. Based on this I-V characteristic, *find the value of* R_{special}.

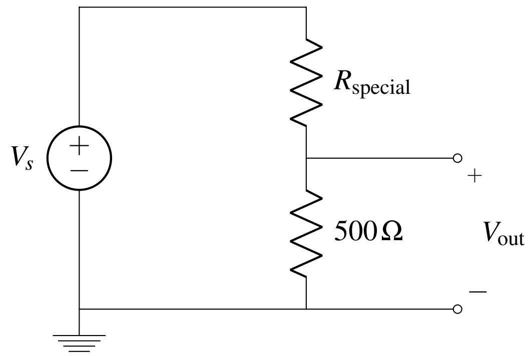


Solution:

$$R_{\text{special}} = \frac{\Delta V}{\Delta I} = \frac{1\,\text{V}}{-0.001\,\text{A}} = -1\,\text{k}\Omega$$

(b) (5 Points) As shown below, Bob draws a voltage divider circuit using R_{special}, a $500\,\Omega$ resistor, and a constant voltage source V_s on a sheet of paper.

Find V_{out} in terms of V_s using your value of R_{special} from part (a).

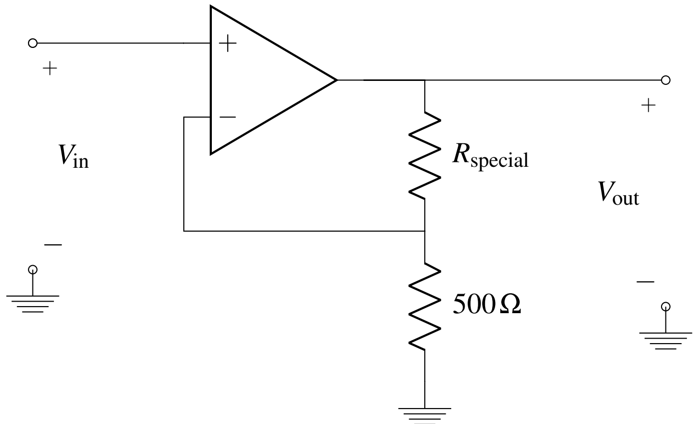


Solution:

Recognizing this is a voltage divider,

$$V_{\text{out}} = \frac{500\,\Omega}{500\,\Omega + R_{\text{special}}} V_s = \frac{500\,\Omega}{500\,\Omega - 1\,\text{k}\Omega} V_s = -V_s$$

(c) (6 Points) Bob now uses the divider with an op-amp in a non-inverting amplifier configuration as shown below. *Is the op-amp below in positive or negative feedback? Make sure to justify your answer.*



Solution:

The op-amp is in positive feedback.

Let's assume that V_{out} increases. Using our result from part (b), we know that V^- decreases. The op-amp outputs $A(V^+ - V^-)$, so the output of the op-amp will increase. Since V_{out} will continue to increase, the op-amp is in positive feedback.

(d) (Bonus: 5 Points) Bob actually builds the circuit from part (c), and he finds that $V_{out} = 3V_{in}$. Based on this result, *did Bob measure $R_{special}$ correctly? Briefly justify your answer.*

Solution:

No. If $R_{special}$ is $1K\Omega$, the opamp above would be in negative feedback. Moreover it's output would be $1 + \frac{R_{special}}{500} V_{in} = 3V_{in}$ Thus Bob did not measure $R_{special}$ correctly.