EECS 16A Spring 2020 — Designing Information Devices and Systems I — Homework 7

This homework is due March 13, 2020 at 23:59.
Self-grades are due March 16, 2020, at 23:59.

Submission Format
Your homework submission should consist of **one** file.

- `hw7.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

1. 1-D Resistive Touchscreen

Figure 1 shows the top view of a resistive touchscreen consisting of a conductive layer with resistivity ρ_{t1}, thickness t, width W, and length L. At the top and bottom it is connected to good conductors ($\rho = 0$), represented in the figure by two rectangles. The touchscreen is wired to voltage source V_s.

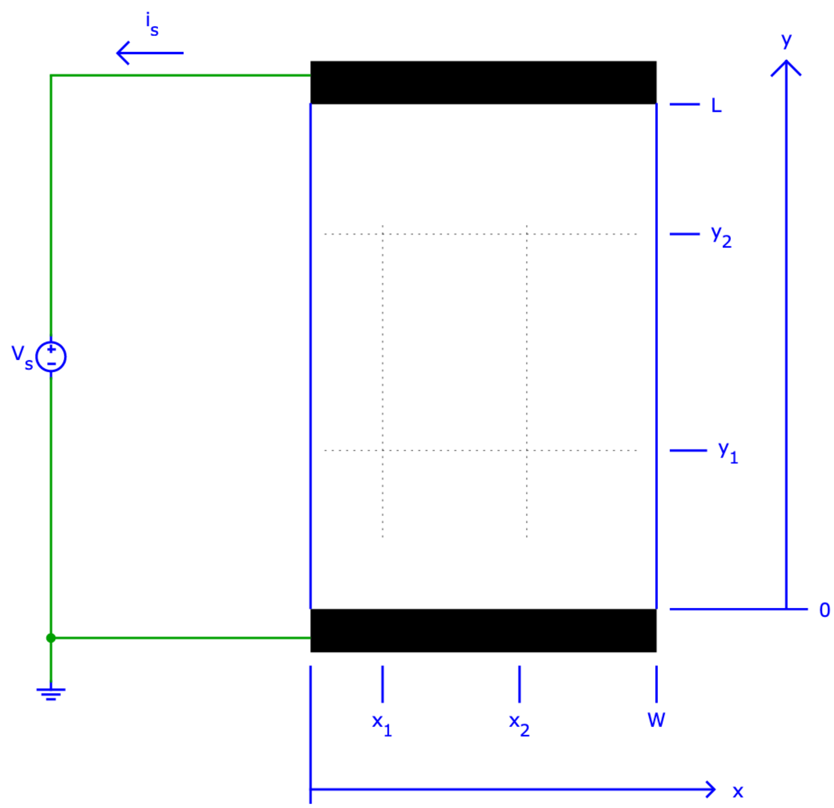
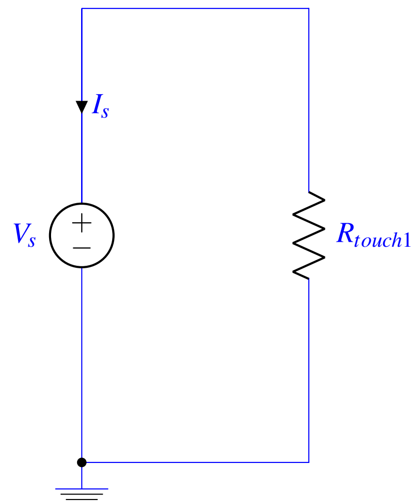


Figure 1: Top view of 1-D resistive touchscreen (not to scale).

Use the following numerical values in your calculations: $W = 50$ mm, $L = 80$ mm, $t = 1$ mm, $\rho_{t1} = 0.5\,\Omega\,\text{m}$, $V_s = 5\text{V}$, $x_1 = 20$ mm, $x_2 = 45$ mm, $y_1 = 30$ mm, $y_2 = 60$ mm.

(a) Draw a circuit diagram representing the touchscreen shown in Figure 1. Remember that circuit diagrams consist of only circuit elements (resistors, current sources, etc) represented by symbols, connecting wires, and the reference symbol.

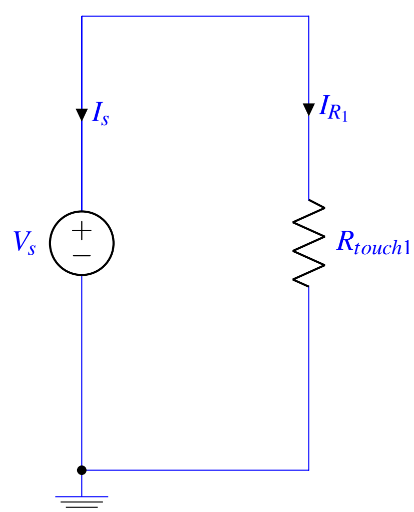
Solution:



(b) Calculate the value of current I_s. Do not forget to specify the correct unit as always.

Solution: The touchscreen resistance can be found from the following expression:

$$\begin{aligned}
R_{touch1} &= \rho_{t1} \cdot \frac{L}{A} \\
&= \rho_{t1} \cdot \frac{L}{W \cdot t} \\
&= 0.5\,\Omega\,\text{m} \left(\frac{80 \times 10^{-3}\,\text{m}}{50 \times 10^{-3}\,\text{m} \cdot 1 \times 10^{-3}\,\text{m}} \right) \\
R_{touch1} &= 800\,\Omega
\end{aligned}$$



From KCL, we can write:

$$I_s + I_{R_1} = 0 \quad (1)$$
$$I_s = -I_{R_1} \quad (2)$$

Therefore, the current I_{R_1} is equal to:

$$I_{R_1} = \frac{V_s}{R_{touch1}} = \frac{5}{800}A = 6.25mA$$

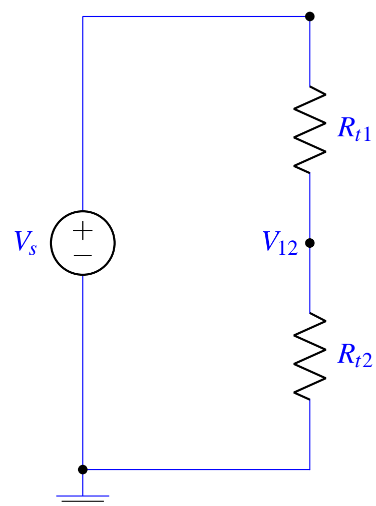
And the current I_s is equal to:

$$I_s = -I_{R_1} = -6.25mA$$

(c) What is the node voltage V_{12} (with respect to the reference node) of the touchscreen at coordinates (x_1, y_2)? Redraw the circuit diagram from part (a) to include node V_{12}. Specify all component values (resistances, ...) in the diagram. Hint: you need more than one resistor to represent this situation.

Solution:

We can represent this setup with the circuit shown below.



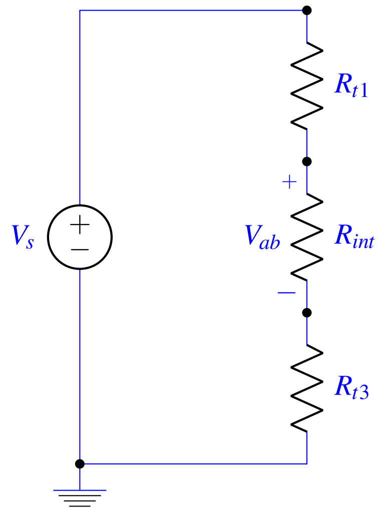
Using voltage division, V_{12} can be found from the following expression:

$$V_{12} = V_s \frac{R_{t2}}{R_{t1} + R_{t2}} = V_s \frac{y_2}{L} = 5 \cdot \frac{3}{4} V = 3.75V$$

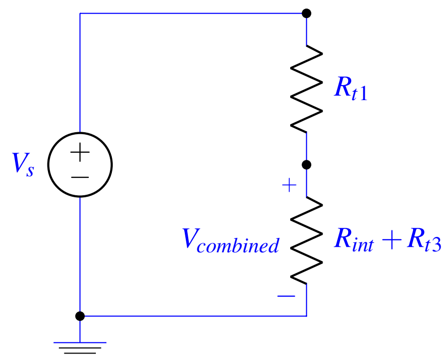
where $R_{t1} = \rho_{t1} \cdot \frac{L-y_2}{Wt}$ and $R_{t2} = \rho_{t1} \cdot \frac{y_2}{Wt}$.

(d) Calculate (absolute value of) voltage V_{ab} between touchscreen coordinates (x_1, y_1) and coordinates (x_1, y_2). Suggestion: Draw an augmented circuit diagram and calculate all component values.

Solution:



One method to find the voltage is by using node voltage analysis. Presented is an alternative approach by using resistor equivalence. To find the voltage, V_{ab}, first, find the voltage over R_{int} and R_{t3} together. We can represent them as an equivalent resistance as follows:



As this circuit is a voltage divider, we can find $V_{combined}$ by voltage division.

$$V_{combined} = \frac{R_{int} + R_{t3}}{R_{t1} + R_{int} + R_{t3}} V_s$$

Once we know $V_{combined}$, this will be the voltage over the two resistors, R_{int} and R_{t3}. We can apply voltage division again to get V_{ab}:

$$V_{ab} = \frac{R_{int}}{R_{int} + R_{t3}} V_{combined}$$

By substituting, we get that V_{ab} is:

$$\begin{aligned} V_{ab} &= \frac{R_{int}}{R_{int} + R_{t3}} \frac{R_{int} + R_{t3}}{R_{t1} + R_{int} + R_{t3}} V_s \\ &= \frac{R_{int}}{R_{t1} + R_{int} + R_{t3}} V_s \end{aligned}$$

Each of the resistances can be calculated as $R_{t1} = \rho_{t1} \cdot \frac{L-y_2}{Wt}$, $R_{int} = \rho_{t1} \cdot \frac{y_2-y_1}{Wt}$ and $R_{t3} = \rho_{t1} \cdot \frac{y_1}{Wt}$. This gives for V_{ab}:

$$V_{ab} = \frac{y_2 - y_1}{L} Vs = \cdot \frac{3}{8} 5V = 1.875V$$

(e) Calculate (absolute value of) the voltage between touchscreen coordinates (x_1, y_1) and coordinates (x_2, y_1).

Solution:

The two points have the same y coordinate, therefore they have the same potential. Thus, $\Delta V = 0$

(f) Calculate (absolute value of) the voltage between touchscreen coordinates (x_1, y_1) and coordinates (x_2, y_2).

Solution:

The two points have different x and y coordinates. However, the potential is the same across the x axis for a fixed y coordinate. Therefore, the problem is similar to part d, since the potential is only determined by the y coordinate of a point. Using the same equivalent circuit of part d we have:

$$\Delta V = V_s \frac{R_{int}}{R_{t1} + R_{int} + R_{t3}} = 1.875V$$

(g) Figure 2 shows a new arrangement with two touchscreens. The second touchscreen is identical to the one shown in Figure 1, except for different width, W_2, and resistivity, ρ_{t2}. Use the following numerical values in your calculations: $W = 50$ mm, $L = 80$ mm, $t = 1$ mm, $\rho_{t1} = 0.5\,\Omega$m, $V_s = 5$V, $x_1 = 20$ mm, $x_2 = 45$ mm, $y_1 = 30$ mm, $y_2 = 60$ mm, which are the same values as before, with $W_2 = 70$ mm, $\rho_{t2} = 0.4\,\Omega$m for the new touchscreen.

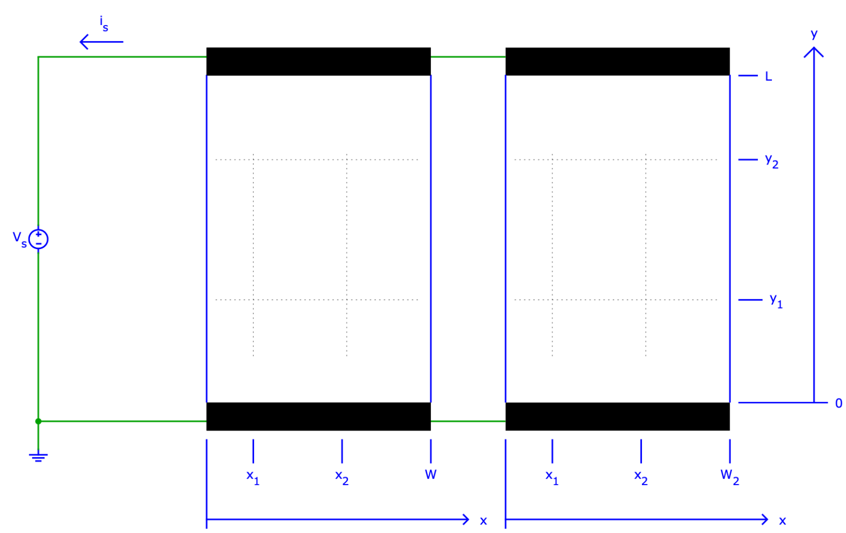
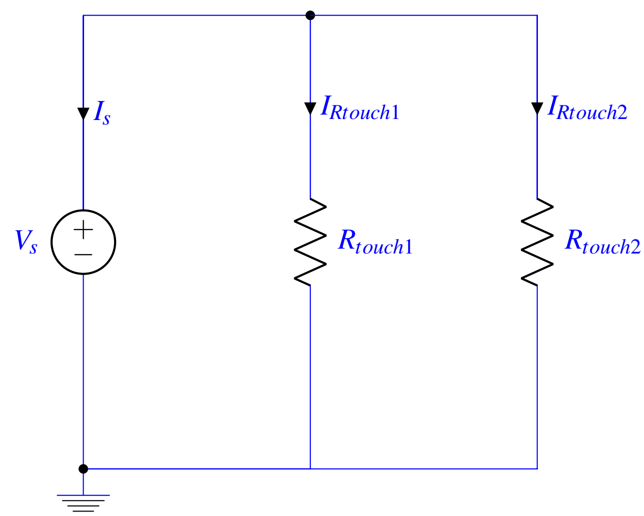


Figure 2: Top view of two touchscreens wired in parallel (not to scale).

Draw a circuit diagram representing the two touchscreens shown in Figure 2.

Solution:



(h) Calculate the value of current I_s for the two touchscreen arrangement.

Solution:

From KCL, we can write:

$$I_s + I_{Rtouch1} + I_{Rtouch2} = 0 \tag{3}$$
$$I_s = -(I_{Rtouch1} + I_{Rtouch2}) \tag{4}$$

Using Ohm's Law for each element:

$$I_s = -\left(\frac{V_s}{R_{touch1}} + \frac{V_s}{R_{touch2}}\right)$$

However, the resistance of the second touchscreen can be given by:

$$R_{touch2} = \rho_{t2} \cdot \frac{L}{W_2 t} = 0.4\,\Omega\,\text{m}\left(\frac{80 \times 10^{-3}\,\text{m}}{70 \times 10^{-3}\,\text{m} \cdot 1 \times 10^{-3}\,\text{m}}\right)$$
$$R_{touch2} = 457.142\,857\,\Omega$$

Therefore, using the resistance values for the first and second touchscreen and the applied voltage source, we have:

$$I_s \approx -(6.25\text{mA} + 10.94\text{mA}) = -17.19\text{mA}$$

(i) Now assume a wire is connected between coordinates (x_1, y_2) in the touchscreen on the left, and (x_2, y_2) in the touchscreen on the right. Calculate the current I_{12} flowing through this wire.

Solution:

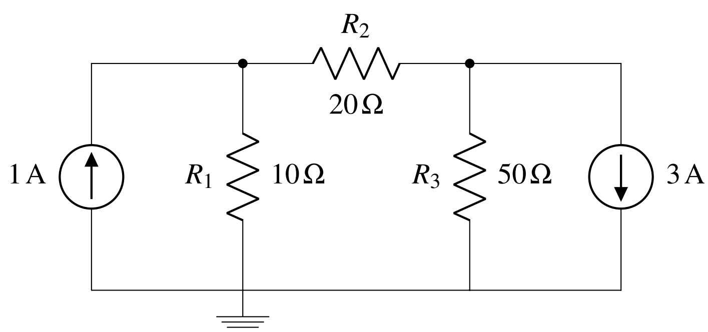
There is no current flowing through the wire, since the points in the two touchscreens have the same potential. Therefore,

$$I_{wire} = \frac{V_{12_{touch1}} - V_{22_{touch2}}}{R_{wire}} = 0$$

We should note that what causes current to flow is a voltage difference. Here, we have a voltage difference of zero, so the current flowing through the wire is zero. Generally, in a wire when current is flowing we have a very small voltage difference $V_{wire} = IR_{wire}$. However, since R_{wire} is super small we neglect V_{wire}.

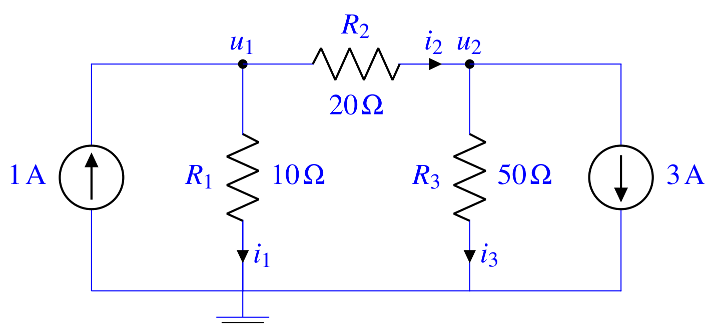
2. Circuit Analysis

Solve the circuit given below for all the currents and all the node voltages.



Solution:

The circuit above only has 3 nodes, one of which is marked as ground. We begin by labeling all of the branch currents and all of the node potentials.



We see that there are 3 unknown currents, i_1, i_2, i_3 and two unknown node potentials u_1, and u_2. Now we write the KCL equations for each of the nodes in the circuit, except for the ground node. For node 1:

$$1\,\mathrm{A} = i_1 + i_2$$

For node 2:

$$i_2 = i_3 + 3\,\mathrm{A}$$

Next we write all the element equations:

$$(u_1 - 0) = V_{R_1}, V_{R_1} = i_1 R_1 \implies (u_1 - 0) = i_1 R_1 \implies i_1 = \frac{u_1}{R_1}$$
$$(u_1 - u_2) = V_{R_2}, V_{R_2} = i_2 R_2 \implies (u_1 - u_2) = i_2 R_2 \implies i_2 = \frac{u_1 - u_2}{R_2}$$
$$(u_2 - 0) = V_{R_3}, V_{R_3} = i_3 R_3 \implies (u_2 - 0) = i_3 R_3 \implies i_3 = \frac{u_2}{R_3}$$

Substituting the branch current expressions into the KCL equations, we have the following equations:

$$1\,\mathrm{A} = \frac{u_1}{R_1} + \frac{u_1 - u_2}{R_2}$$
$$\frac{u_1 - u_2}{R_2} = \frac{u_2}{R_3} + 3\,\mathrm{A}$$

We can re-express the system of equations in the following way:

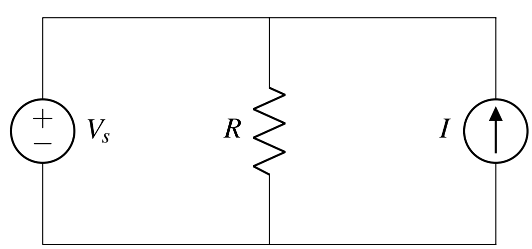
$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1\,\mathrm{A} \\ -3\,\mathrm{A} \end{bmatrix}$$

Plugging in the values for all of the resistors, and solving, we find that $u_1 = -10\,\mathrm{V}$ and $u_2 = -50\,\mathrm{V}$.

We can find the currents by using our Ohm's law's equations:

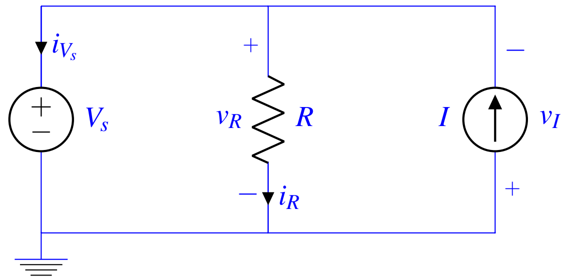
$$i_1 = \frac{-10\,\mathrm{V}}{10\,\Omega} = -1\,\mathrm{A}$$
$$i_2 = \frac{-10\,\mathrm{V} - (-50\,\mathrm{V})}{20\,\Omega} = 2\,\mathrm{A}$$
$$i_3 = \frac{-50\,\mathrm{V}}{50\,\Omega} = -1\,\mathrm{A}$$

3. Power Analysis



(a) Find the power dissipated by each element in the circuit above. Remember to label voltages using passive sign convention.

Solution: We label a ground node, and then solve for the currents i_V, i_R and the voltages V_R, V_I.



Solving the above circuit using nodal analysis, we get

$$i_R = \frac{V_s}{R}$$

$$i_V = I - \frac{V_s}{R}$$

$$v_I = -V_s$$

$$v_R = V_s$$

Using this we can calculate

$$P_{V_s} = V_s i_V = IV_s - \frac{{V_s}^2}{R}$$

$$P_I = Iv_I = -IV_s$$

$$P_R = i_R v_R = \frac{{V_s}^2}{R}$$

Note that $P_{V_s} + P_I + P_R = 0$, i.e. energy provided is energy dissipated, which verifies our intuition about conservation of energy.

(b) Use $R = 5\text{k}\Omega$, $V_s = 5\text{V}$, and $I = 5\text{mA}$. Calculate P_{V_s}, P_I, and P_R.

Solution:

$P_{V_S} = (0.005\text{A})(5\text{V}) - \frac{(5\text{V})^2}{5000\Omega} = 0.02W$
$P_I = -(0.005\text{A})(5\text{V}) = -0.025\text{W}$
$P_R = \frac{(5\text{V})^2}{5000\Omega} = 0.005\text{W}$

Note that $P_{V_S} + P_I + P_R = 0$.

(c) Repeat part (b) but change the value I of the current source such that it **dissipates** 40mW. Calculate I, P_{V_s}, P_I, and P_R.

Solution:

Remember that using passive sign convention, an element whose power is negative is supplying power, and an element whose power is positive is dissipating power. Therefore, we want $P_I = 40\text{mW}$. We know that $P_I = -IV_S$. Therefore, $I = \frac{0.04\text{W}}{-5\text{V}} = -0.008\text{A}$.

$P_{V_S} = (-0.008\text{A})(5\text{V}) - \frac{(5\text{V})^2}{5000\Omega} = -0.045W$
$P_I = -(-0.008\text{A})(5\text{V}) = 0.04\text{W}$
$P_R = \frac{(5\text{V})^2}{5000\Omega} = 0.005\text{W}$

Note that $P_{V_S} + P_I + P_R = 0$.

4. Cell Phone Battery

As great as smartphones are, one of their drawbacks is that their batteries don't last a long time. For example, a Google Pixel phone, under typical usage conditions (internet, a few cat videos, etc.) uses 0.3W. We will model the battery as an ideal voltage source (which maintains a constant voltage across its terminals regardless of current) except that we assume that the voltage drops abruptly to zero when the battery is discharged (in reality the voltage drops gradually, but let's keep things simple).

Battery capacity is specified in mAh, which indicates how many mA of current the battery can supply for one hour before it needs to be recharged. The Pixel's battery has a battery capacity of 2770mAh at 3.8V. For example, this battery could provide 1000mA (or 3.8W) for 2.77 hours before the voltage abruptly drops from 3.8V to zero.

(a) How long will a Pixel's full battery last under typical usage conditions?

Solution:

300 mW of power at 3.8 V is about 79 mA of current. Our 2770 mAh battery can supply 79 mA for $\frac{2770\,\text{mAh}}{79\,\text{mA}} = 35\,\text{h}$, or about a day and a half.

(b) How many coulombs of charge does the battery contain? How many usable electrons worth of charge are contained in the battery when it is fully charged? (An electron has $1.602 \times 10^{-19}\,\text{C}$ of charge.)

Solution:

One hour has 3600 seconds, so the battery's capacity can be written as $2770\,\text{mAh} \times 3600\,\frac{\text{s}}{\text{h}} = 9.972 \times 10^6\,\text{mAs} = 9972\text{As} = 9972\text{C}$.

An electron has a charge of approximately $1.602 \times 10^{-19}\,\text{C}$, so 9972 C is $\frac{9972\,\text{C}}{1.602 \times 10^{-19}\,\text{C}} \approx 6.225 \times 10^{22}$ electrons. That's a lot!

(c) Suppose the cell phone battery is completely discharged and you want to recharge it completely. How much energy (in J) is this? Recall that a J is equivalent to a W s.

Solution:

The battery capacity is 2770 mAh at 3.8 V, which means the battery has a total stored energy of $2770\,\text{mAh} \cdot 3.8\,\text{V} = 10.5\,\text{Wh} = 10.5\text{Wh} \cdot 3600\text{s} = 37.9\text{kJ}$.

(d) Suppose PG&E charges \$0.12 per kWh. Every day, you completely discharge the battery (meaning more than typical usage) and you recharge it every night. How much will recharging cost you for the month of October (31 days)?

Solution:

2770 mAh at 3.8 V is $2770\,\text{mAh} \cdot 3.8\,\text{V} = 10.5\,\text{Wh}$, or 0.01 kWh. At \$0.12 per kWh, that is $\$0.12 \cdot 0.01$ per day, or $\$0.12 \cdot 0.01 \cdot 31 = \$0.037$, or about 4 cents a month. Compare that to your cell phone data bill! Whew!

(e) The battery has internal circuitry that prevents it from getting overcharged (and possibly exploding!). We will model the battery and its internal circuitry as a resistor R_{bat}. We now wish to charge the battery by plugging into a wall plug. The wall plug can be modeled as a 5 V voltage source and 200 mΩ resistor, as pictured in Figure 3. What is the power dissipated across R_{bat} for $R_{\text{bat}} = 1\,\text{m}\Omega$, $1\,\Omega$, and $10\,\text{k}\Omega$? (i.e. how much power is being supplied to the phone battery as it is charging?). How long will the battery take to charge for each of those values of R_{bat}?

Solution:

The energy stored in the battery is 2770 mAh at 3.8 V, which is $2.77\,\text{Ah} \cdot 3.8\,\text{V} = 10.5\,\text{Wh}$. We can find the time to charge by dividing this energy by power in W to get time in hours.

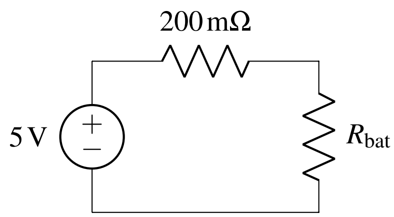


Figure 3: Model of wall plug, wire, and battery.

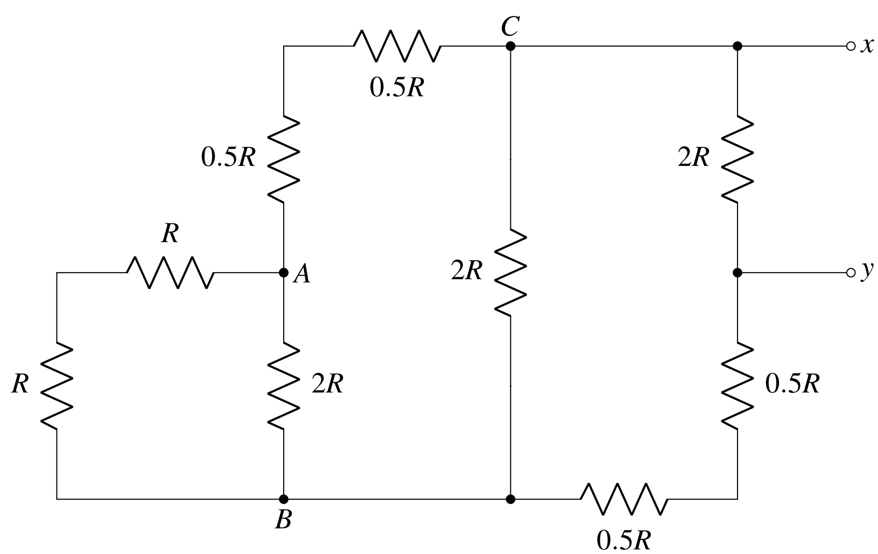
For $R_{\text{bat}} = 1\,\text{m}\Omega$:
The total resistance seen by the battery is $1\,\text{m}\Omega + 200\,\text{m}\Omega = 201\,\text{m}\Omega$ (because the wire and R_{bat} are in series), so by Ohm's law, the current is $\frac{5\,\text{V}}{0.201\,\Omega} = 24.88\,\text{A}$. The voltage drop across R_{bat} is (again by Ohm's law) $24.88\,\text{A} \cdot 0.001\,\Omega = 0.02488\,\text{V}$. Then power is $0.02488\,\text{V} \cdot 24.88\,\text{A} = 0.619\,\text{W}$, and the total time to charge the battery is $\frac{10.5\,\text{Wh}}{0.619\,\text{W}} = 17\,\text{h}$.

For $R_{\text{bat}} = 1\,\Omega$:
The total resistance seen by the battery is $1\,\Omega + 0.2\,\Omega = 1.2\,\Omega$, the current through the battery is $\frac{5\,\text{V}}{1.2\,\Omega} = 4.167\,\text{A}$, and the voltage across the battery is by Ohm's law $4.167\,\text{A} \cdot 1\,\Omega = 4.167\,\text{V}$. Then the power is $4.167\,\text{A} \cdot 4.167\,\text{V} = 17.36\,\text{W}$, and the total time to charge the battery is $\frac{10.5\,\text{Wh}}{17.36\,\text{W}} = 0.6\,\text{h}$, about $36\,\text{min}$.

For $R_{\text{bat}} = 10\,\text{k}\Omega$:
The total resistance seen by the battery is $10000\,\Omega + 0.2\,\Omega = 10000.2\,\Omega$, the current through the battery is $\frac{5\,\text{V}}{10000.2\,\Omega} \approx 0.5\,\text{mA}$, and the voltage across the battery is by Ohm's law $0.5\,\text{mA} \cdot 10\,\text{k}\Omega \approx 5\,\text{V}$ (up to 2 significant figures). Then the power is $5\,\text{V} \cdot 0.5\,\text{mA} = 2.5\,\text{mW}$, and the total time to charge the battery is $\frac{10.526\,\text{Wh}}{0.0025\,\text{W}} = 4210\,\text{h}$.

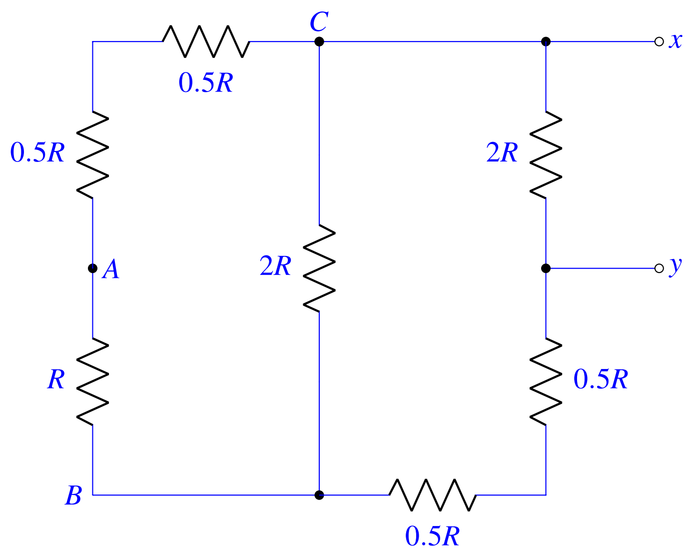
5. Equivalence

For the circuit shown below, find the equivalent resistance looking in from points x and y.

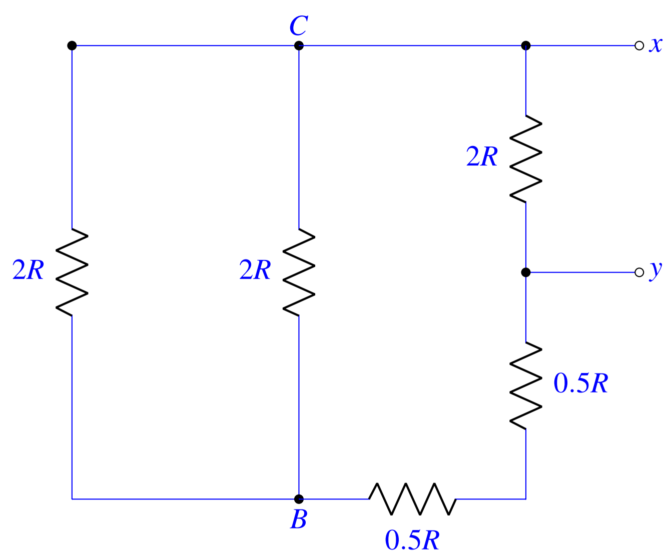


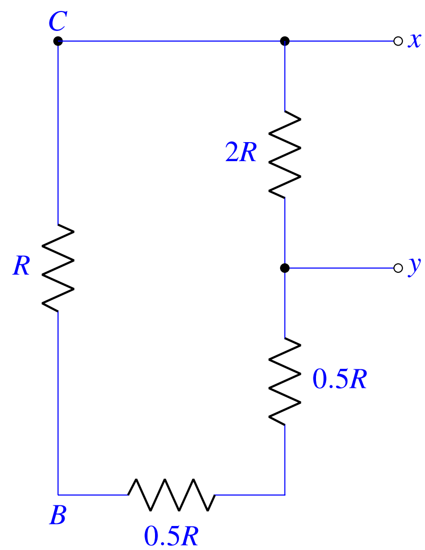
Solution:

We find the equivalent resistance for the resistors from left to right. First we find R_{AB}.

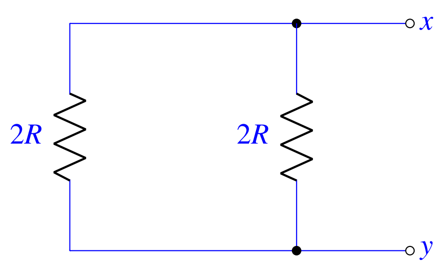


Then we can find R_{CB}.

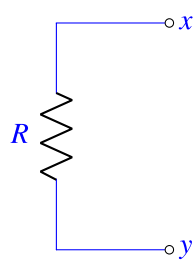




If we move from node C counter-clockwise to node y, the resistance seen is $R+0.5R+0.5R=2R$. Therefore we have,



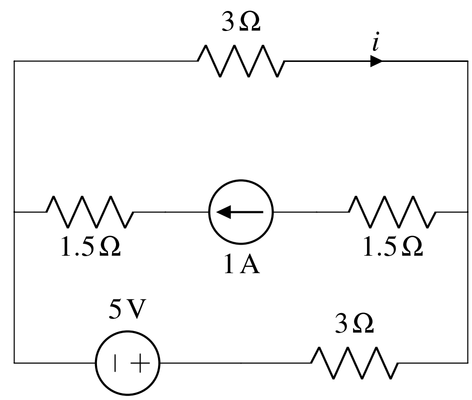
Now we can find R_{xy}.



Therefore, the equivalent resistance is $R_{xy}=R$.

6. Superposition

Find the current i indicated in the circuit diagram below using superposition.

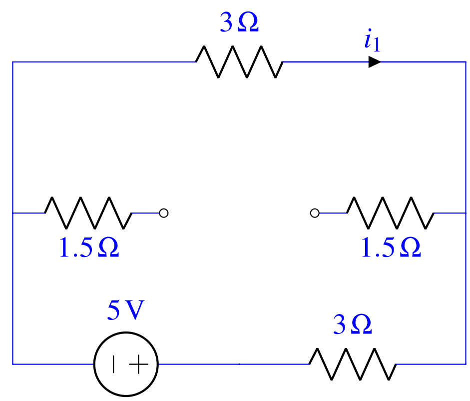


Solution:

$$i = -\frac{1}{3}\,\mathrm{A}$$

Consider the circuits obtained by:

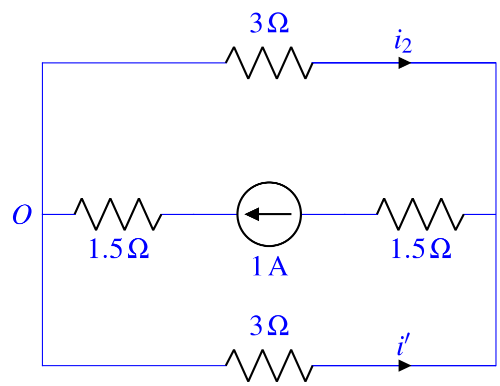
(a) Turning off the 1 A current source:



In the above circuit, no current is going to flow through the middle branch, as it is an open circuit. Thus this is just a 5 V voltage source connected to two 3 Ω resistors in series so

$$i_1 = -\frac{5}{6}\,\mathrm{A}$$

(b) Turning off the 5 V voltage source:



In the above circuit, notice that the $3\,\Omega$ resistors are in parallel and therefore form a current divider. Since the values of the resistances are equal, the current flowing through them will also be equal, that is $i_2 = i'$. Applying KCL to node O, we get

$$1 - i_2 - i' = 0$$

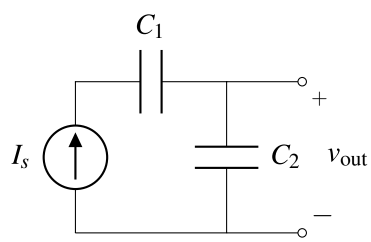
which gives us

$$i_2 = \frac{1}{2}\,\mathrm{A}$$

Now, applying the principle of superposition, we have $i = i_1 + i_2 = -\frac{5}{6}\,\mathrm{A} + \frac{1}{2}\,\mathrm{A} = -\frac{1}{3}\,\mathrm{A}$.

7. Current Sources And Capacitors

For the circuit given below, give an expression for $v_{\text{out}}(t)$ in terms of I_s, C_1, C_2, and t. Assume that all capacitors are initially uncharged, i.e. the initial voltage across each capacitor is $0\,\mathrm{V}$.



Solution:

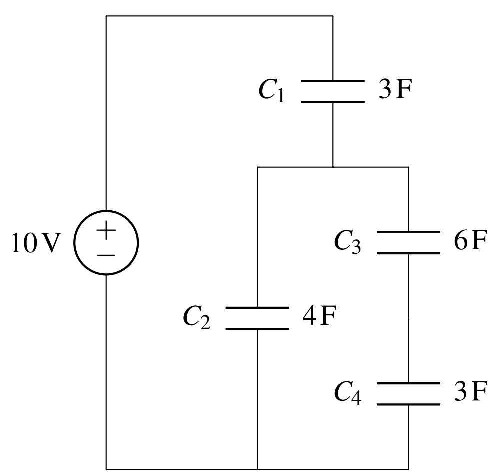
By KCL, the current I_s flowing through C_1 must be the current flowing through C_2. $v_{\text{out}}(0) = 0$ because all capacitors are initially uncharged.

$$I_s = C_2 \frac{dv_{\text{out}}(t)}{dt}$$

$$v_{\text{out}}(t) = \int \frac{I_s}{C_2} dt = \frac{I_s t}{C_2} + v_{\text{out}}(0) = \frac{I_s t}{C_2}$$

8. (Practice) Mechanical Circuits with Capacitors

Find the voltages across and currents flowing through all of the capacitors at steady state.

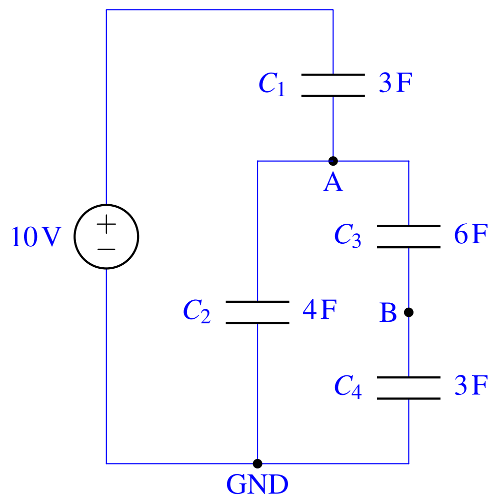


Solution:

For a capacitor C_k, let us denote the voltage across it by v_{C_k}, the current flowing through it by i_{C_k}, and its charge by Q_{C_k}. In steady state (that is, after the current has been running for a very long time), direct current (DC) capacitors act as open circuits. Hence, we see that there is no current flowing through the capacitors, that is,

$$i_{C_1} = i_{C_2} = i_{C_3} = i_{C_4} = 0\,\text{A}.$$

For finding the voltages across the capacitors, let us label nodes on the circuit as shown in the following figure.



We are going to use the following four properties to find the voltages across the capacitors:

(a) Charge is always conserved.

(b) The charge Q stored in a capacitor is given by the equation $Q = CV$.

(c) The charges across series capacitors are equal to each other.

(d) The voltage across parallel capacitors is equal.

As an example use of property (c), we have the charge on the capacitor C_3 equal to the charge on the capacitor C_4.

Let us start by writing the equation for conservation of charge at node A:

$$Q_{C_1} = Q_{C_2} + Q_{C_3}$$

By property (b), that is, $Q = CV$, we can equivalently write this eqaution for charge conservation in terms of node voltages as

$$(10\,\text{V} - v_A)3\,\text{F} = v_A 4\,\text{F} + (v_A - v_B)6\,\text{F},$$

which, after simplifiying the equation, gives

$$30\,\text{V} = 13v_A - 6v_B. \tag{5}$$

Let us then write the charge conservation equation at node B; we have

$$Q_{C_3} = Q_{C_4}.$$

As before, we can write this charge conservation equation in terms of the node voltages as

$$(v_A - v_B)6\,\mathrm{F} = v_B 3\,\mathrm{F},$$

which, after simplification, gives

$$2v_A = 3v_B. \tag{6}$$

Equations 5 and 6 give us two linearly independent equations in two unknowns. Solving the system, we get

$$v_A = \frac{10}{3}\,\mathrm{V},$$
$$v_B = \frac{20}{9}\,\mathrm{V}.$$

Using the node voltages, we can calculate the voltages across the capacitors as

$$v_{C_1} = 10\,\mathrm{V} - v_A = \frac{20}{3}\,\mathrm{V},$$
$$v_{C_2} = v_A = \frac{10}{3}\,\mathrm{V},$$
$$v_{C_3} = v_A - v_B = \frac{10}{9}\,\mathrm{V},$$
$$v_{C_4} = v_B = \frac{20}{9}\,\mathrm{V}.$$

We write the currents across the capacitors again here for reader's convenience:

$$i_{C_1} = i_{C_2} = i_{C_3} = i_{C_4} = 0\,\mathrm{A}$$

9. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.) How did you work on this homework?

Solution:

I worked on this homework with...

I first worked by myself for 2 hours, but got stuck on problem 5, so I went to office hours on...

Then I went to homework party for a few hours, where I finished the homework.