
EECS 16A Designing Information Devices and Systems I

Spring 2020 Homework 6

This homework is due March 6, 2020, at 23:59.

Self-grades are due March 9, 2020, at 23:59.

Submission Format

Your homework submission should consist of **one** file.

- `hw6.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned) as well as your IPython notebook saved as a PDF.

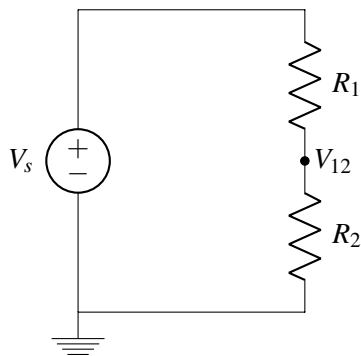
If you do not attach a PDF “printout” of your IPython notebook, you will not receive credit for problems that involve coding. Make sure that your results and your plots are visible. Assign the IPython printout to the correct problem(s) on Gradescope.

Submit the file to the appropriate assignment on Gradescope.

1. Voltage divider

In the following parts, $V_s = 10\text{ V}$. Choose resistance values such that the current through each element is $\leq 1\text{ A}$.

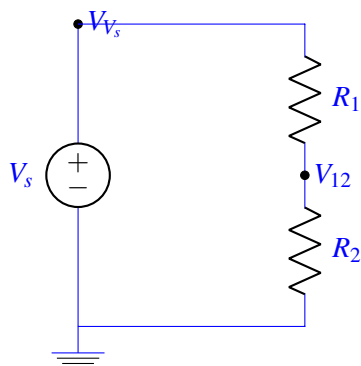
- (a) Select values for R_1 and R_2 in the circuit below such that $V_{12} = 5\text{ V}$.



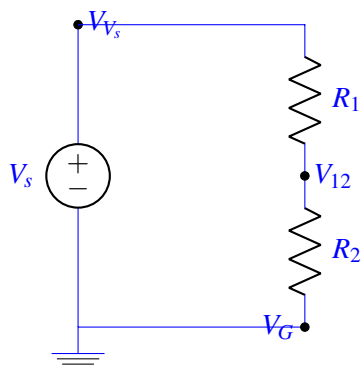
Solution: Step 1: Reference Node

We notice that the ground node has already been selected for us in the question.

Step 2: Label nodes with voltage set by sources



Step 3: Label other nodes



Step 4: Label element voltages and currents.

Let V_1, V_2 be the voltage drop across R_1 and R_2 respectively. Let I_1 be the current between the voltage source and R_1 . Let I_2 be the current from R_2 to the voltage source.

Step 5: KCL Equations

From KCL, at V_{12} ,

$$I_1 = I_2.$$

Step 6: Find element currents

Using Ohm's law, we have that

$$I_1 = \frac{V_1}{R_1}$$

$$I_2 = \frac{V_2}{R_2}$$

Writing the element voltages in terms of the node voltages, we have that $V_1 = V_s - V_{12}$ and $V_2 = V_{12}$.

Step 7: Substitute element currents into KCL Equations

Substituting back, we obtain $\frac{V_s - V_{12}}{R_1} = \frac{V_{12}}{R_2}$. Solving, we find that $V_{12} = \frac{R_2}{R_1 + R_2} V_s$. Plugging in $V_{12} = 5V$ and $V_s = 10V$, we see that $R_1 = R_2$ must be true.

To choose R_1 and R_2 such that the current through each element is $\leq 1A$, use KVL to write an expression for I_1, I_2 as a function of R_1, R_2 :

$$V_s - I_1 R_1 - I_2 R_2 = 0, \text{ with } I_1 = I_2 = I_s$$

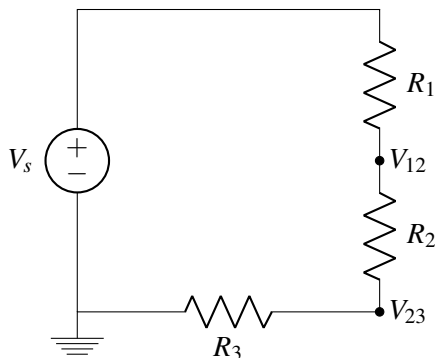
$$V_s = I_s(R_1 + R_2)$$

$$10V = 1A(R_1 + R_2)$$

$$R_1 + R_2 = 10\Omega$$

As $R_1 + R_2$ must be at least 10Ω , and $R_1 = R_2$, we choose $R_1 = R_2 = 5\Omega$.

(b) Select values for R_1, R_2, R_3 in the circuit below such that $V_{12} = 5V$ and $V_{23} = 3V$.

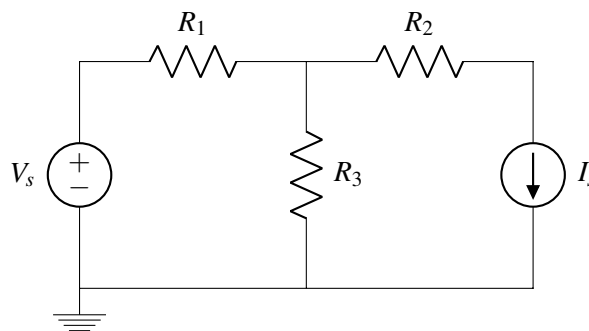


Solution: Let V_1, V_2, V_3 be the voltage drop across R_1, R_2, R_3 respectively. From KCL, we know that the current flowing through each of the elements is the same. Using Ohm's law, this can be rewritten as $\frac{V_1}{R_1} = \frac{V_2}{R_2} = \frac{V_3}{R_3}$. Writing the element voltages in terms of the node voltages, we have that $V_1 = V_s - V_{12}$ and $V_2 = V_{12} - V_{23}$ and $V_3 = V_{23}$. Solving, similarly to the previous subpart, we find that $V_{12} = \frac{R_2 + R_3}{R_1 + R_2 + R_3} V_s$ and $V_{23} = \frac{R_3}{R_1 + R_2 + R_3} V_s$. We found in the previous subpart that the total resistance in our KVL loop must be at least 10Ω to limit the current to $\leq 1A$, meaning for this circuit that $R_1 + R_2 + R_3 \geq 10\Omega$. Choosing $R_1 = 5\Omega, R_2 = 2\Omega, R_3 = 3\Omega$ would give us the desired voltage, and limit the current to $\leq 1A$.

2. Circuit Analysis

Using the steps outlined in lecture, solve the following circuits for the currents through each branch and the voltages at each node. Use the ground node labelled for you. You may use a numerical tool, such as IPython.

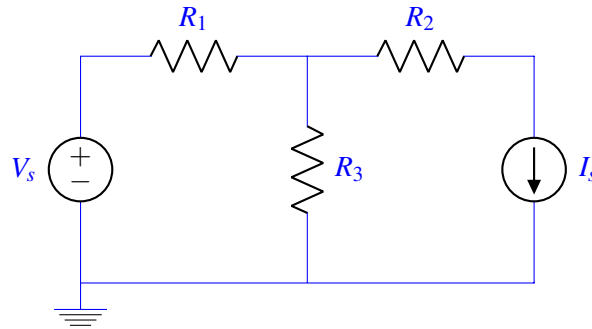
(a) $V_s = 5V, I_s = 2A, R_1 = R_2 = 2\Omega, R_3 = 4\Omega$



Solution:

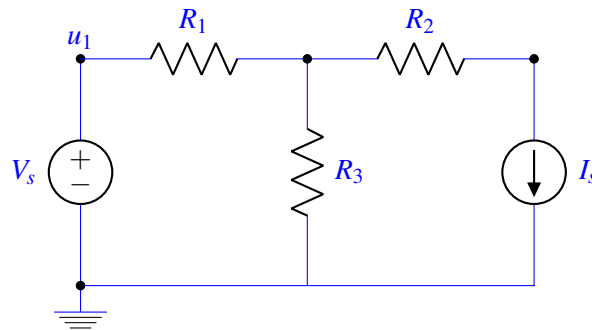
Step 1) Reference node

Select a reference (ground) node. Any node can be chosen for this purpose. In this example, we choose the node at the bottom of the circuit diagram.



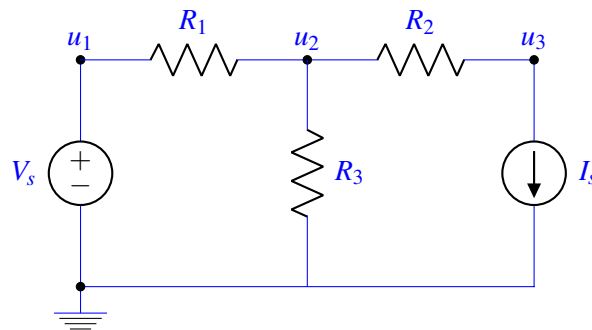
Step 2) Label Nodes with Voltage set by Sources

Voltage sources set the voltage of the node they are connected to. In the example, there is only one source, V_s , and we label the corresponding source u_1 (names are arbitrary, but must be unique).



Step 3) Label Remaining Nodes

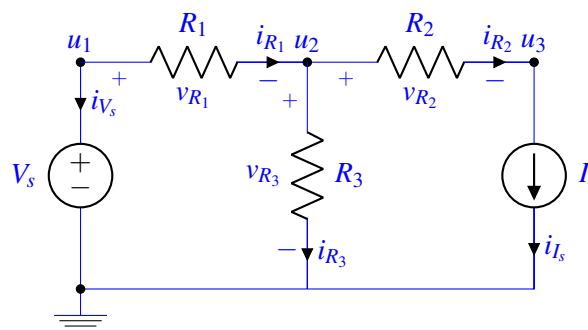
Now we label all remaining nodes in the circuit except the reference. In the example there are two, u_2 and u_3 .



Step 4) Label Element Voltages and Currents

Next we mark all element voltages and currents.

Start with the current. The direction is arbitrary (top to bottom, bottom to top, it won't matter, but stick with your choice in subsequent steps). Then mark the element voltages following the passive sign convention, i.e. the voltage and current point in the "same" direction.



Step 5) KCL Equations

Write KCL equations for all nodes with unknown voltage, u_2 and u_3 .

$$\begin{aligned} -i_{R_1} + i_{R_2} + i_{R_3} &= 0 \\ -i_{R_2} + i_{I_s} &= 0 \end{aligned}$$

Step 6) Element Currents

Find expressions for all element currents in terms of voltage and element characteristics (e.g. Ohm's law) for all circuit elements except voltage sources. In this circuit there are four, R_1 , R_2 , R_3 and I_s .

$$\begin{aligned} i_{I_s} &= I_s \\ i_{R_1} &= \frac{V_{R_1}}{R_1} \\ i_{R_2} &= \frac{V_{R_2}}{R_2} \\ i_{R_3} &= \frac{V_{R_3}}{R_3} \end{aligned}$$

We also have

$$\begin{aligned} u_1 &= V_s \\ i_{I_s} &= I_s \\ i_{R_1} &= \frac{u_1 - u_2}{R_1} \\ i_{R_2} &= \frac{u_2 - u_3}{R_2} \\ i_{R_3} &= \frac{u_2}{R_3} \end{aligned}$$

Step 7) Substitute Element Currents in KCL Equations

Now we substitute the expressions derived in Step 6 into the KCL equations from Step 5.

$$-\frac{V_s - u_2}{R_1} + \frac{u_2 - u_3}{R_2} + \frac{u_2}{R_3} = 0$$

$$-\frac{u_2 - u_3}{R_2} + I_s = 0$$

Let's make this a bit nicer by grouping the unknowns (u_2 and u_3) on the left side and the known terms on the right:

$$u_2 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) + u_3 \left(-\frac{1}{R_2} \right) = \frac{V_s}{R_1}$$

$$u_2 \left(-\frac{1}{R_2} \right) + u_3 \left(\frac{1}{R_2} \right) = -I_s$$

Note that we now have 2 equations for 2 unknowns. Thus, we set up the following matrix relation:

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \frac{V_s}{R_1} \\ -I_s \end{bmatrix}$$

Finally, we plug in the values we were given into the matrix above and use Gaussian elimination to find the vector of unknowns. We find that:

$$\begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0.667V \\ -3.33V \end{bmatrix}$$

The branch currents can be obtained from the node voltages and element equations. Therefore, we can write:

$$i_{V_s} = -2.167A$$

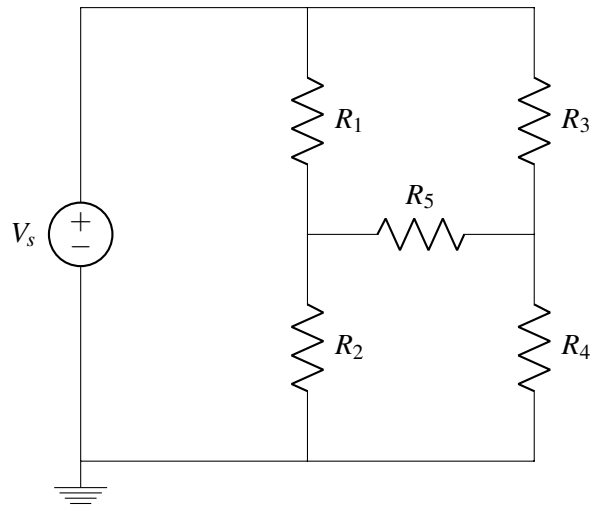
$$i_{I_s} = I_s = 2A$$

$$i_{R_1} = \frac{u_1 - u_2}{R_1} = 2.167A$$

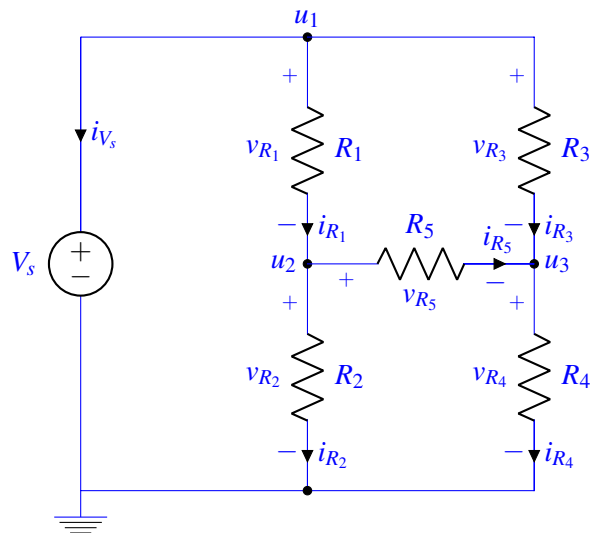
$$i_{R_2} = \frac{u_2 - u_3}{R_2} = 2A$$

$$i_{R_3} = \frac{u_2}{R_3} = 0.167A$$

(b) $V_s = 5V, R_1 = 1\Omega, R_2 = 2\Omega, R_3 = 3\Omega, R_4 = 4\Omega, R_5 = 5\Omega$

**Solution:**

Here, we will skip showing all of the individual steps. Below is the circuit with our choice of ground and current directions.



From the above circuit, we get the following KCL equations:

$$-i_{R1} + i_{R2} + i_{R5} = 0$$

$$-i_{R3} + i_{R4} - i_{R5} = 0$$

Using the IV relations for each element, we have:

$$\begin{aligned} u_1 - 0 &= V_s \\ i_{R_1} &= \frac{u_1 - u_2}{R_1} \\ i_{R_2} &= \frac{u_2 - 0}{R_2} \\ i_{R_3} &= \frac{u_1 - u_3}{R_3} \\ i_{R_4} &= \frac{u_3}{R_4} \\ i_{R_5} &= \frac{u_2 - u_3}{R_5} \end{aligned}$$

We also know that $u_1 = V_s$

Now we substitute these expressions into the KCL equations we previously derived.

$$\begin{aligned} -\frac{V_s - u_2}{R_1} + \frac{u_2 - 0}{R_2} + \frac{u_2 - u_3}{R_5} &= 0 \\ -\frac{V_s - u_3}{R_3} + \frac{u_3}{R_4} - \frac{u_2 - u_3}{R_5} &= 0 \end{aligned}$$

Let's make this a bit nicer by grouping the unknowns (u_2 and u_3) on the left side and the known terms on the right:

$$\begin{aligned} u_2 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5} \right) + u_3 \left(-\frac{1}{R_5} \right) &= \frac{V_s}{R_1} \\ u_2 \left(-\frac{1}{R_5} \right) + u_3 \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \right) &= \frac{V_s}{R_3} \end{aligned}$$

Note that we now have 2 equations for 2 unknowns. Thus, we set up the following matrix relation:

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5} & -\frac{1}{R_5} \\ -\frac{1}{R_5} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \frac{V_s}{R_1} \\ \frac{V_s}{R_3} \end{bmatrix}$$

Finally, we plug in the values we were given into the matrix above and use Gaussian elimination to find the vector of unknowns.

$$\begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 3.29V \\ 2.968V \end{bmatrix}$$

The branch currents can be obtained from the node voltages and element equations. Therefore, we can write:

$$i_{R_1} = \frac{u_1 - u_2}{R_1} = 1.709A$$

$$i_{R_2} = \frac{u_2 - 0}{R_2} = 1.645A$$

$$i_{R_3} = \frac{u_1 - u_3}{R_3} = 0.677A$$

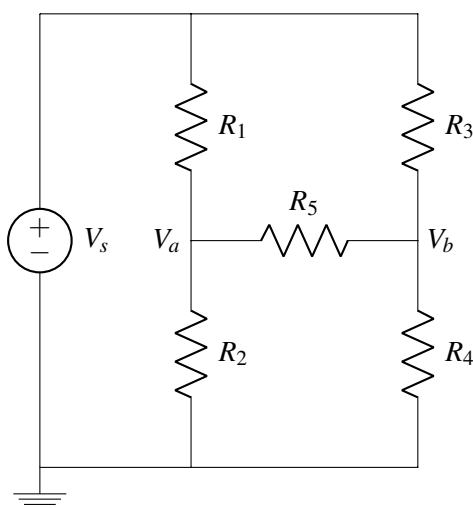
$$i_{R_4} = \frac{u_3}{R_4} = 0.741A$$

$$i_{R_5} = \frac{u_2 - u_3}{R_5} = 0.0644A$$

$$i_{V_s} = i_{R_1} + i_{R_3} = -2.38A$$

3. Volt and ammeter

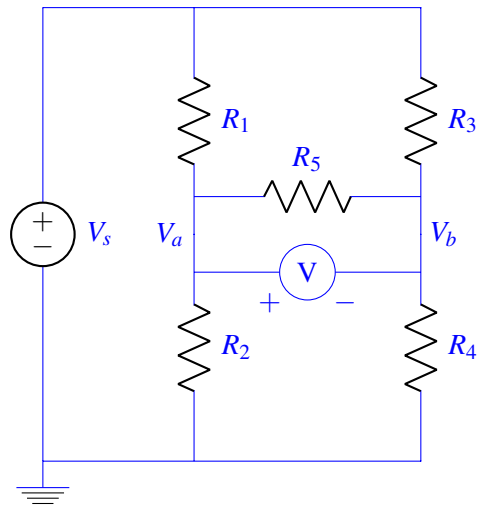
Use the following numerical values in your calculations: $R_1 = 1\text{ k}\Omega$, $R_2 = 2\text{ k}\Omega$, $R_3 = 3\text{ k}\Omega$, $R_4 = 4\text{ k}\Omega$, $R_5 = 5\text{ k}\Omega$, $V_s = 5V$.



- (a) Redraw the circuit diagram shown in Figure 1 by adding a voltmeter (letter V in a circle and plus and minus signs indicating direction) to measure voltage V_{ab} from node V_a (positive) to node V_b (negative). Calculate the value of V_{ab} .

Hint: You have analyzed a very similar circuit elsewhere in this assignment, reuse the result.

Solution: Below is the redrawn circuit with the voltmeter. Note that it is also correct to have the voltmeter above R_5 , as it will still be connected to the same nodes.



Using the same analysis with part b from Problem 1, we end up with the following matrix relation for the node voltages V_a and V_b :

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5} & -\frac{1}{R_5} \\ -\frac{1}{R_5} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} \frac{V_s}{R_1} \\ \frac{V_s}{R_3} \end{bmatrix}$$

Plugging in the values we were given into the matrix above and using Gaussian elimination we can find the vector of unknowns.

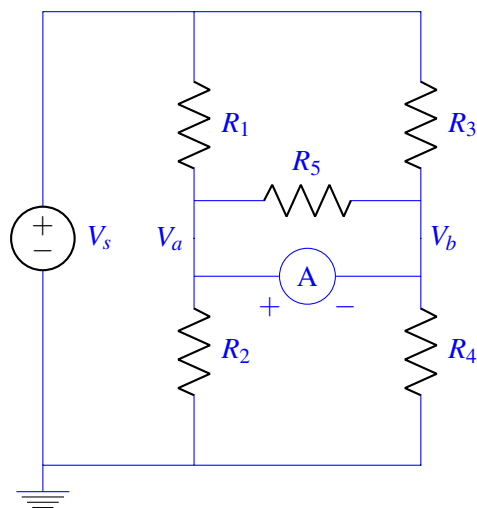
$$\begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} 3.29V \\ 2.968V \end{bmatrix}$$

From these node voltages, the voltage V_{ab} can be calculated.

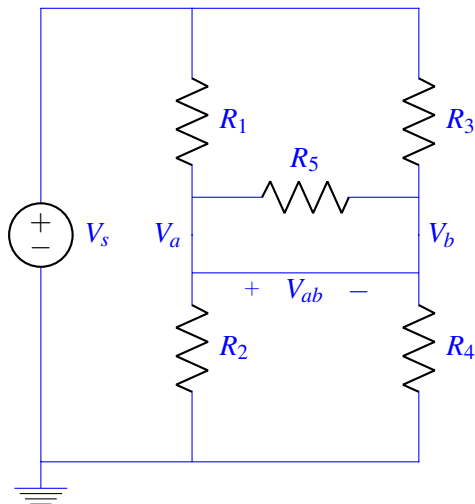
$$V_{ab} = V_a - V_b = 0.322V$$

- (b) Suppose you inadvertently connected an ammeter in part (a) above, rather than a voltmeter (we all goof sometimes). Calculate the value of V_{ab} with the ammeter connected. Note: it differs from the value calculated in part (a).

Solution: While you did not have to redraw the circuit, it is depicted below.

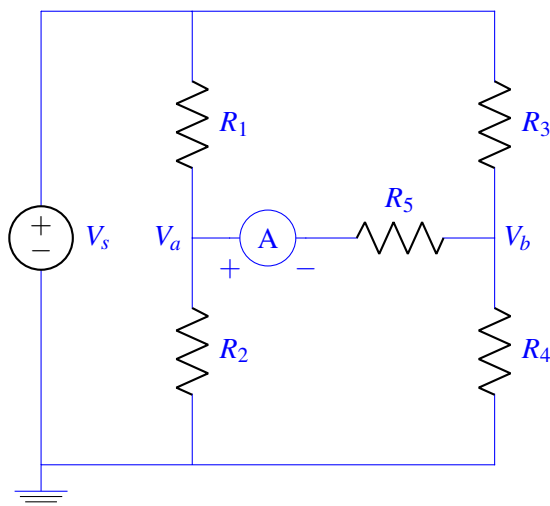


If we assume that the internal resistance of an ammeter is ideally zero, placing it across the nodes V_a and V_b will short them. So $V_a = V_b$. Thus $V_{ab} = 0$. The circuit below shows how the ammeter behaves as a short that unifies the previously separate nodes.



- (c) Redraw the circuit diagram shown in Figure 1 by adding an ammeter (letter A in a circle and plus and minus signs indicating direction) to measure the current I_{R_5} through resistor R_5 . Calculate the value of I_{R_5} .

Solution: The redrawn circuit with the ammeter measuring the current through R_5 is shown in the following circuit. It is also correct to draw the ammeter to the right of R_5 with the orientation of the meter remaining the same: the plus sign should be most proximal to the node labeled V_a , and the minus sign is most proximal to the node labeled V_b .

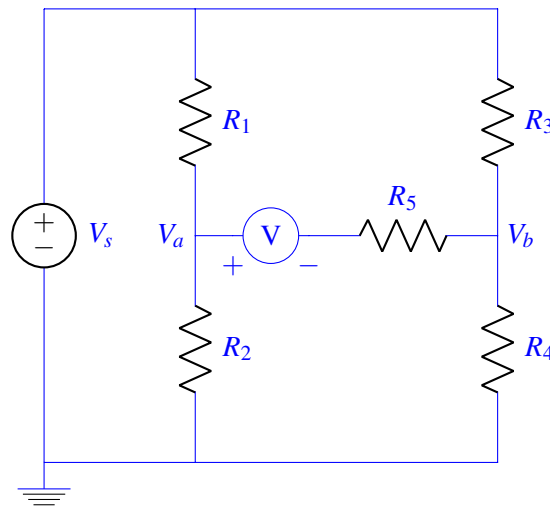


After calculating the node voltages V_a and V_b from part a, we can write:

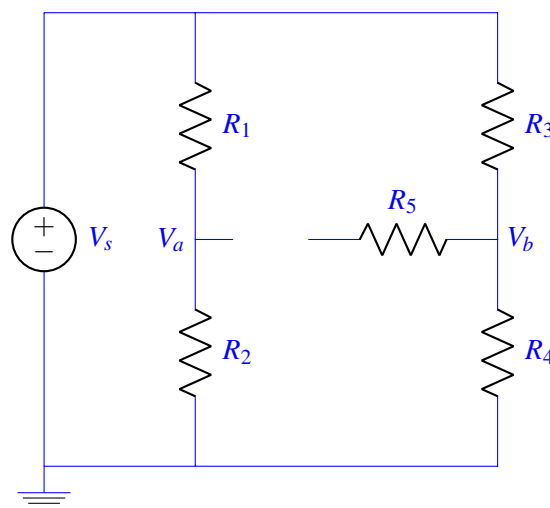
$$I_{R_5} = \frac{V_a - V_b}{R_5} = 64.4 \mu A$$

- (d) Your friend inadvertently connected a voltmeter in part (c) above, rather than an ammeter. Calculate the value of I_{R_5} with the voltmeter connected. Note: it differs from the value calculated in part (c).

Solution: While you were not required to redraw the new circuit, the circuit is shown bellow.



The resistance of a voltmeter is infinite and it behaves as an open circuit. There will be no current flowing through R_5 . Therefore, $I_{R_5} = 0$. The circuit below depicts how the voltmeter behaves as an open that prevents any current through R_5 .

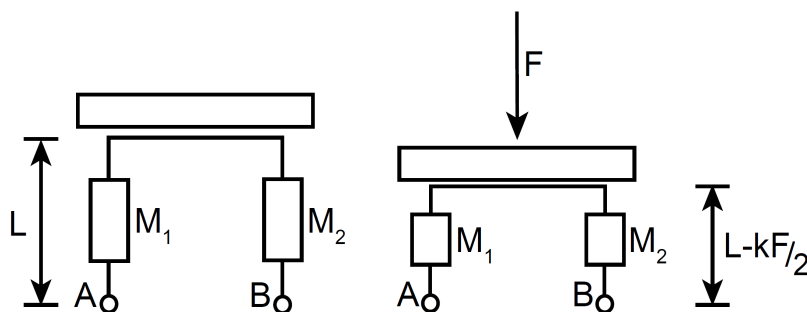


4. Fruity Fred

Fruity Fred just got back from Berkeley Bowl with a bunch of mangoes, pineapples, and coconuts. He wants to sort his mangoes in order of weight, so he decides to use his knowledge from EECS16A to build a scale.

He finds two identical bars of material (M_1 and M_2) of length L (meters) and cross-sectional area A_c (meters²), which are made of a material with resistivity ρ . He knows that the length of these bars decreases by k meters per Newton of force applied, while the cross-sectional area remains constant.

He builds his scale as shown below, where the top of the bars are connected with an ideal electrical wire. The left side of the diagram shows the scale at rest (with no object placed on it), and the right side shows it when the applied force is F (Newtons), causing the length of each bar to decrease by $kF/2$ meters. Fred's mangoes are not very heavy, so $L \gg kF/2$.



- (a) Let R_{AB} be the resistance between nodes A and B. Write an expression for R_{AB} as a function of A_c , L , ρ , F , and k .

Solution:

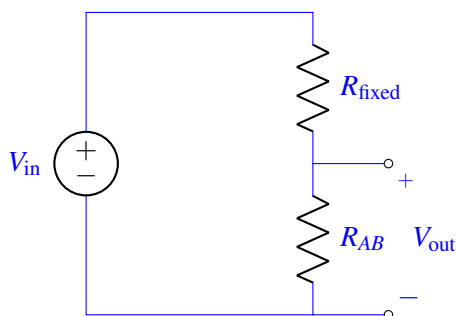
Note that, the length of each bar decreases by k meters per newton of force applied it, and since there are two bars supporting the scale, each bar can be thought to have half the force, $\frac{F}{2}$ on it. Because of this, each bar's length diminishes by $\frac{kF}{2}$, to have lengths of $L - \frac{kF}{2}$.

The combination of R_1 and R_2 has a resistance $R_{AB} = R_1 + R_2$, which is $R_{AB} = \frac{2\rho(L - \frac{kF}{2})}{A_c}$

- (b) Fred's scale design is such that the resistance R_{AB} changes depending on how much weight is placed on it. However, he really wants to measure a voltage rather than a resistance.

Design a circuit for Fred that outputs a voltage that is some function of the weight. Your circuit should include R_{AB} , and you may use any number of voltage sources and resistors in your design. Be sure to label where the voltage should be measured in your circuit. Also provide an expression relating the output voltage of your circuit to the force applied on the scale.

Solution: If you utilized either of the resistances, R_{AB} indicated in the previous subpart, that is acceptable. One possible solution: use a voltage divider.



$$V_{\text{out}} = \frac{R_{AB}}{R_{\text{fixed}} + R_{AB}} V_{\text{in}}$$

$$V_{\text{out}} = \frac{\frac{2\rho(L - \frac{kF}{2})}{A_c}}{R_{\text{fixed}} + \frac{2\rho(L - \frac{kF}{2})}{A_c}} V_{\text{in}} = \frac{2\rho(L - \frac{kF}{2})}{R_{\text{fixed}}A_c + 2\rho(L - \frac{kF}{2})} V_{\text{in}}$$

5. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.) How did you work on this homework?

Solution:

I worked on this homework with...

I first worked by myself for 2 hours, but got stuck on problem 5, so I went to office hours on...

Then I went to homework party for a few hours, where I finished the homework.