EECS 16A Designing Information Devices and Systems I
Spring 2020 Homework 1

This homework is due Friday January 31, 2020, at 23:59.
Self-grades are due Monday February 3, 2020, at 23:59.

Please note that future homeworks will have more problems than HW1 and we ask you to plan accordingly.

Submission Format

Your homework submission should consist of a single PDF file that contains all of your answers (any handwritten answers should be scanned) as well as your IPython notebook saved as a PDF.

Please attach a PDF of your Jupyter notebook for all the problems that involve coding. Make sure the results of your plots (if any) are visible. Please assign the PDF of the notebook to the correct problems on Gradescope — we will be unable to grade the problems without this assignment or submission.

Homework Learning Goals: *The objective of this homework is to introduce systems of linear equations. This homework additionally serves as an introduction to working with the Python environment through IPython/Jupyter notebooks.*

1. Counting Solutions

Learning Goal: *(This problem is meant to illustrate the different types of systems of equations. Some have a unique solution and others have no solutions or infinitely many solutions. We will learn in this class how to systematically figure out which of the three above cases holds.)*

For each of the following systems of linear equations, determine if there is a unique solution, no solution, or an infinite number of solutions. If there is a unique solution, find it. If there is an infinite number of solutions state this and give one solution. If there is no solution, explain why. **Use augmented matrices and show your work**.

(a)

$$\begin{aligned} 2x + 3y &= 5 \\ x + y &= 2 \end{aligned}$$

(b)

$$\begin{aligned} x + y + z &= 3 \\ 2x + 2y + 2z &= 5 \end{aligned}$$

(c)

$$\begin{aligned} - y + 2z &= 1 \\ 2x \quad + z &= 2 \end{aligned}$$

(d)

$$\begin{aligned} x + 2y &= 3 \\ 2x - y &= 1 \\ 3x + y &= 4 \end{aligned}$$

(e)

$$\begin{aligned} x &+ 2y &= 3 \\ 2x &- y &= 1 \\ x &- 3y &= -5 \end{aligned}$$

2. Dutta's Optimal Boba

Learning Goal: *Recognize a problem that can be cast as a systems of linear equations.*

Dutta's Optimal Boba has a unique way of serving its customers. To ensure the best customer experience, each customer gets a combination drink personalized to their tastes. Professor Dutta knows that a lot of customers don't know what they want, so when customers walk up to the counter, they are asked to taste four standard combination drinks that each contain a different mixture of the available pure teas.

Each combination drink (Classic, Roasted, Mountain, and Okinawa) is made of a mixture of pure teas (Black, Oolong, Green, and Earl Grey), with the total amount of pure tea in each combination drink always the same, and equal to one cup. The table below shows the quantity of each pure tea (Black, Oolong, Green, and Earl Grey) contained in each of the four standard combination drinks (Classic, Roasted, Mountain, and Okinawa).

Tea [cups]	Classic	Roasted	Mountain	Okinawa
Black	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{2}{3}$
Oolong	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{1}{3}$
Green	0	$\frac{1}{3}$	$\frac{3}{5}$	0
Earl Grey	$\frac{1}{3}$	0	0	0

Initially, the customer's ratings for each of the pure teas are unknown. Professor Dutta's goal is to determine how much the customer likes each of the pure teas, so that an optimal combination drink can then be made. By letting the customer taste and score each of the four standard combination drinks, Professor Dutta can use linear algebra to determine the customer's initially unknown ratings for each of the pure teas. After a customer gives a score (all of the scores are real numbers) for each of the four standard combination drinks, Professor Dutta then calculates how much the customer likes each pure tea and mixes up a special combination drink that will maximize the customer's score.

The score that a customer gives for a combination drink is a linear combination of the ratings of the constituent pure teas, based on their proportion. For example, if a customer's rating for black tea is 6 and oolong tea is 3, then the total score for the Okinawa boba drink would be $6 \cdot \frac{2}{3} + 3 \cdot \frac{1}{3} = 5$ because Okinawa has $\frac{2}{3}$ black tea and $\frac{1}{3}$ Oolong tea.

Professor Courtade was thirsty after giving the first lecture, so Professor Courtade decided to take a drink break at Dutta's Optimal Boba. Professor Courtade walked in and gave the following ratings:

Combination Drink	**Score**
Classic	7
Roasted	7
Mountain	$7\frac{2}{5}$
Okinawa	$6\frac{1}{3}$

(a) What were Professor Courtade's ratings for each tea? **Work this problem out by hand in terms of the steps. You may use a calculator to do algebra.**

(b) What mystery tea combination could Professor Dutta put in Professor Courtade's personalized drink to maximize the customer's score? If there is more than one correct answer, state that there are many answers, and give one such combination. What score would Professor Courtade give for the answer you wrote down?

3. Filtering Out The Troll

Learning Goal: *(The goal of this problem is to represent a practical scenario using a simple model of directional microphones. Students will tackle the problem of sound reconstruction through solving a system of linear equations.)*

You attended a very important public speech and recorded it using a recording device that consists of two directional microphones. However, there was this particular person in the audience who was trolling around, adding noise to the recording. When you went back home to listen to the recording, you realized that the two recordings were dominated by the troll's noise and you could not hear the speech. Fortunately, since your recording device contained two microphones, you realized there is a way to combine the two individual microphone recordings so that the troll's noise is removed. You remembered the locations of the speaker and the troll and created the diagram shown in Figure 1. You (and your two microphones) are located at the origin.

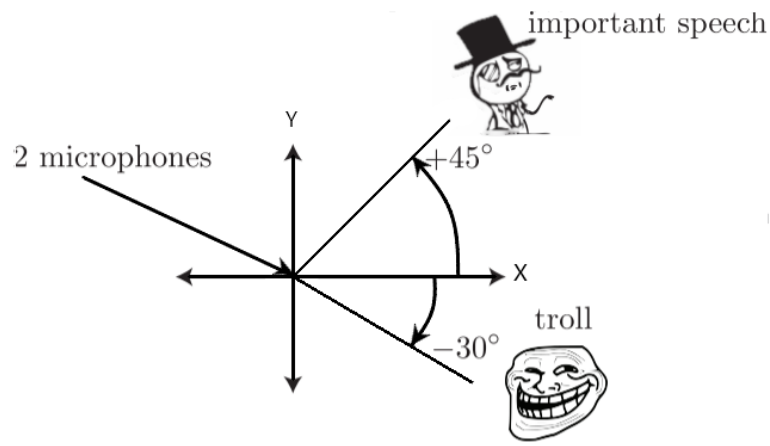

Figure 1: Locations of the speaker and the troll.

Each directional microphone records signals differently based on where they are coming from. For the first microphone, when a signal is coming from an angle θ with respect to the x-axis, it is weighted by the factor $f_1(\theta) = \cos(\theta)$. If there are two signals simultaneously playing (as is the case with the speech and the troll noise), then both are recorded as a linear combination, each weighted by the respective $f_1(\theta)$ for their angles). For the second microphone, if the signal is coming from an angle θ with respect to the x-axis, then the signal is weighted by the factor $f_2(\theta) = \sin(\theta)$. The linear combination also applies to the second microphone. Graphically, the directional characteristics of the two microphones are given in Figure 2.

We can now refer to the diagram in Figure 1 and develop a mathematical model of the microphone recordings. Let the person who gave the important speech and the troll be speakers A and B, respectively. The person who gave the important speech (speaker A) was located at angle $\alpha = +45°$ relative to the x-axis, and the troll (speaker B) was located at angle $\beta = -30°$ relative to the x-axis. Speaker A produced an audio signal represented by the vector $\vec{a} \in \mathbb{R}^n$. That is, the i-th entry of vector $\vec{a}$ was the signal at the i-th time step.

Similarly, speaker B produced an audio signal $\vec{b} \in \mathbb{R}^n$, where the i-th entry of vector $\vec{b}$ was the signal at the i-th time step.

Therefore, the first microphone recorded the signal

$$\vec{m}_1 = f_1(\alpha) \cdot \vec{a} + f_1(\beta) \cdot \vec{b},$$

and the second microphone recorded the signal

$$\vec{m}_2 = f_2(\alpha) \cdot \vec{a} + f_2(\beta) \cdot \vec{b}.$$

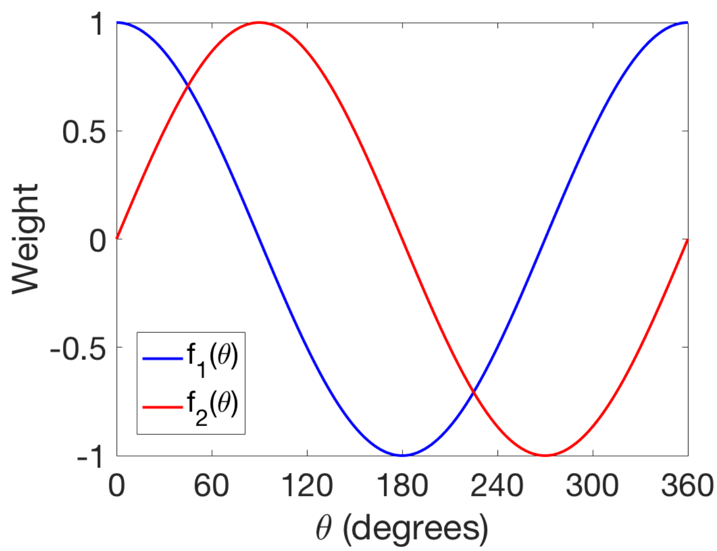

Figure 2: Weights for recorded audio signals for each of the two microphones, as a function of audio source angle θ. Microphone 1 is blue and microphone 2 is red. Note that a weight can be negative as well as positive.

(a) Using the notation above, express the recordings of the two microphones $\vec{m}_1$ and $\vec{m}_2$ (i.e. the signals recorded by the first and the second microphones, respectively) as a linear combination (i.e. a weighted sum) of $\vec{a}$ and $\vec{b}$.

(b) Recover the important speech $\vec{a}$, as a weighted combination of $\vec{m}_1$ and $\vec{m}_2$. In other words, write $\vec{a} = u \cdot \vec{m}_1 + v \cdot \vec{m}_2$ (where u and v are scalars). What are the values of u and v?

(c) Partial IPython code can be found in `prob1.ipynb`. Complete the code to get the signal of the important speech. Write out what the speaker says. (Optional: Where is the speech taken from?)

Note: You may have noticed that the recordings of the two microphones sound remarkably similar. This means that you could recover the real speech from two "trolled" recordings that sound almost identical! Leave out the fact that the recordings are actually different, and have some fun with your friends who aren't lucky enough to be taking EECS16A.

4. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.) How did you work on this homework?

