This homework is due Friday, July 29, 2022 at 23:59.
Self-grades are due Monday, August 1, 2022 at 23:59.

Submission Format
Your homework submission should consist of one file.

1. Reading Assignment (OPTIONAL)

For this homework, please read Notes 16, 17 and 17B. Note 16 will provide an introduction to capacitors (a circuit element which stores charge), capacitive equivalence, and the underlying physics behind them. Note 17 will provide an overview of the capacitive touchscreen, comparator, and Op-Amps. Note 17B will provide a walkthrough of the charge-sharing algorithm.

(a) Describe the key ideas behind how a capacitor works. How are capacitor equivalences calculated? Compare this with how we calculate resistor equivalences.

(b) Consider the capacitive touchscreen. Describe how it works, and compare and contrast it to the resistive touchscreens we have seen in previous lectures and homeworks.

(c) In the charge sharing algorithm, what property of charge is applied in connecting phase 1 calculations to phase 2 calculations?

(d) If the op-amp supply voltages are $V_{DD} = 5\, \text{V}$ and $V_{SS} = 0\, \text{V}$, then what is the minimum/maximum value of $V_{out}$?

(e) What is the purpose of a comparator? How can we use a comparator circuit to detect a touch for a capacitive touchscreen?

2. It's finally raining!

A lettuce farmer in Salinas Valley has grown tired of weather.com’s imprecise rain measurements. Therefore, they decided to take matters into their own hands by building a rain sensor. They placed a rectangular tank outside and attached two metal plates to two opposite sides in an effort to make a capacitor whose capacitance varies with the amount of water inside.
The width and length of the tank are both \( w \) (i.e., the base is square) and the height of the tank is \( h_{\text{tot}} \).

(a) What is the capacitance between terminals \( a \) and \( b \) when the tank is full? What about when it is empty? 
   \( \text{Note: the permittivity of air is } \epsilon, \text{ and the permittivity of rainwater is } 81 \epsilon. \)

(b) Suppose the height of the water in the tank is \( h_{\text{H}_2\text{O}} \). Modeling the tank as a pair of capacitors in parallel, find the total capacitance between the two plates. Call this capacitance \( C_{\text{tank}} \).

(c) After building this capacitor, the farmer consults the internet to assist them with a capacitance-measuring circuit. A fellow internet user recommends the following:

\[
\begin{align*}
I_s & \quad C_{\text{tank}} \quad V_C(t), V_C(0) = 0 \text{V} \\
& \quad I_c \\
\end{align*}
\]

In this circuit, \( C_{\text{tank}} \) is the total tank capacitance that you calculated earlier. \( I_s \) is a known current supplied by a current source.
The suggestion is to measure \( V_C \) for a brief interval of time, and then use the difference to determine \( C_{\text{tank}} \).
Determine \( V_C(t) \), where \( t \) is the number of seconds elapsed since the start of the measurement. You should assume that before any measurements are taken, the voltage across \( C_{\text{tank}} \), i.e. \( V_C \), is initialized to 0 V, i.e. \( V_C(0) = 0 \).

(d) If we can measure \( V_C(t) \) and knowing the result of part (c), how could we derive the value of \( C_{\text{tank}} \)? Then, using the result from part (b), write \( h_{\text{H}_2\text{O}} \) as a function of \( C_{\text{tank}} \) and other constants.

3. Circuit with Capacitors

Find the voltages at nodes \( u_A \) and \( u_B \), and currents flowing through all of the capacitors at steady state. Assume that before the voltage source is applied, the capacitors all initially have a charge of 0 Coulombs.
4. Capacitive Touchscreen

The model for a capacitive touchscreen can be seen in Figure 1. See Table 1 for values of the dimensions. The green area represents the contact area of the finger with the top insulator. It has dimensions $w_2 \times d_1$, where $w_2$ is the horizontal width of the finger contact area and $d_1$ is the depth (into the page) of the finger contact area. The top metal (red area) has dimensions $w_1 \times d_1$. The bottom metal (grey area) has dimensions $w \times d_2$, where $w$ is larger than both $w_1$ and $w_2$. The vertical distance between the top metal (red) and bottom plate (grey) is $t_1$, and the vertical distance between the finger (green) and the bottom plate (grey) is $t_2$.

Table 1: Touchscreen Dimension Values

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<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$d_1$</td>
<td>10 mm</td>
</tr>
<tr>
<td>$d_2$</td>
<td>1 mm</td>
</tr>
<tr>
<td>$t_1$</td>
<td>2 mm</td>
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<tr>
<td>$t_2$</td>
<td>4 mm</td>
</tr>
<tr>
<td>$w_1$</td>
<td>1 mm</td>
</tr>
<tr>
<td>$w_2$</td>
<td>2 mm</td>
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</table>

(a) Draw the equivalent circuit of the touchscreen that contains the nodes $F$, $E_1$, and $E_2$ when: (i) there no finger present; and (ii) when there is a finger present. Express the capacitance values in terms of $C_0$, $C_1$, and $C_2$.

*Hint: Note that node $F$ represents the finger. When there is no touch node $F$ would be non-existent. Hint: Treat $E_1$ as the "top node", $E_2$ as the "bottom node", and the finger $F$ as an intermediate node when present.*

(b) What are the values of $C_0$, $C_1$, and $C_2$? Assume that the insulating material has a permittivity of $\varepsilon = 4.43 \times 10^{-11} F/m$ and that the thickness of the metal layers is small compared to $t_1$ (so you can ignore the thickness of the metal layers). Also assume that the right edge of the top metal (red area) in the diagram is aligned with the right edge of the finger (green area) in the diagram.

(c) What is the difference in effective capacitance between the two metal plates (nodes $E_1$ and $E_2$) when a finger is present?

(d) What are the advantages of capacitive touchscreen over resistive touchscreen?

*Hint: Can we do multi-fingers detections using capacitive touchscreen? What about resistive touchscreen?*
5. Fun With Charge Sharing

(a) Capacitors $C_1$ and $C_2$ are charged to $V_1$ and $V_2$ and switch $S_1$ is open for time $t < 0$. At time $t = 0$, switch $S_1$ is closed. Calculate $V_1$ at time $t > 0$.

(b) The circuit shown below operates in two phases. During phase 1, switches labeled $S_1$ are closed and switches $S_2$ are open. During phase 2, switches $S_1$ are open and switches $S_2$ are closed. Assume that the initial voltage on $C_x = 0$.

Use the following values: $C_1 = 1 \text{F}$, $C_2 = 4 \text{F}$, $V_1 = 6 \text{V}$, $V_2 = 1 \text{V}$.
i. Redraw the circuit during phase 1. Replace closed switches with "wires" and open switches with "open circuits" (i.e. just omit them from the diagram). Use $C_1 = C_2 = C_0$.

ii. Redraw the circuit during phase 2. Replace closed switches with "wires" and open switches with "open circuits" (i.e. just omit them from the diagram). Use $C_1 = C_2 = C_0$.

iii. Calculate the value of the voltage $V_{out}$ during phase 2 as a function of $C_0$, $C_x$, and $V_s$. Use $C_1 = C_2 = C_0$.

6. LED Alarm Circuit

One day, you come back to your dorm to find that your favorite candy has been stolen. Determined to catch the perpetrator red-handed, you decide to put the candy inside a kitchen drawer. Using the following circuit design, you would like to turn on a light-emitting diode (LED) “alarm” if the kitchen drawer is opened.

Note $R_{photo}$ is a photoresistor, which acts like a typical resistor but changes resistance based on the amount of light it is exposed to. This photoresistor is located inside the kitchen drawer, so we can tell when the drawer is opened or closed.
$V_{LED}$ indicates the voltage across the LED; we will guide you through the IV behavior of this element later in the problem. The LED is located in your room (and connected to a long wire going to the kitchen), so that you can remotely tell when the kitchen drawer has been opened.

(a) What is $V_+$, the voltage at the positive voltage input of the comparator? Your answer should be written in terms of $R_{\text{photo}}$ and $R_{\text{fixed}}$.

(b) We now want to choose a value for $R_{\text{fixed}}$. From the photoresistor’s datasheet, we see the resistance in “light” conditions (i.e. drawer open) is 1 kΩ. In “dark” conditions (i.e. drawer closed), the resistance is 10 kΩ.

To ensure the comparator detects the light condition with more tolerance, we decide to design $R_{\text{fixed}}$ so that $V_+$ is 3 V under the “light” condition. Solve for the value of $R_{\text{fixed}}$ to meet this specification.

(c) Write down $V_{\text{out}}$ with any conditions in terms of $V_+$. For simplicity, consider the case when $V_+ \neq V_-$ and assume the comparator is ideal.

(d) Using your answers to the previous parts, write down $V_{\text{out}}$ with the conditions on its output in terms of $R_{\text{photo}}$. You can substitute the value of $R_{\text{fixed}}$ you found in part (b). As before, you can assume that $V_+ \neq V_-$ and the comparator is ideal.

(e) From the design steps in the previous parts, we have designed a circuit that outputs non-zero voltage when the photoresistor is exposed to light (i.e. kitchen drawer open). We now want to design the LED portion of the circuit, so we get a visual alarm when the drawer is open.

From the LED’s datasheet, the forward voltage, $V_F$ is 3 V. Essentially, if $V_{\text{LED}}$ is less than this voltage, the LED won’t light up and $I_{\text{LED}}$ will be 0 A.

Here is an idealized IV curve of this LED. The LED behaves in one of the following two modes:

i. If the voltage across the LED is less than $V_F = 3$ V or if $I_{\text{LED}} < 0$ A, then the LED acts like an open circuit.

ii. If the voltage across the LED is $V_F = 3$ V, then the LED acts like a voltage source, except that it only allows positive current flow (i.e. only in the direction of current marked on the circuit diagram).

![IV Curve of LED](image)

To avoid exceeding the power rating of the LED (and having it burn out), the recommended value for $I_{\text{LED}}$ is 20 mA.

Find the value of the current-limiting resistor, $R_{\text{lim}}$, such that when the photoresistor is in the “light” condition, $I_{\text{LED}} = 20$ mA.
7. **Op-Amp in Negative Feedback**

In this question, we analyze op-amp circuits that have finite op-amp gain \( A \). We replace the op-amp with its circuit model with parameterized gain and observe the gain’s effect on terminal and output voltages as the gain approaches infinity. **Note here that** \( V_{SS} = -V_{DD} \).

![Op-Amp Circuit Diagram]

For parts (a) - (e) only, assume that the op-amp is ideal (i.e., \( A \to \infty \)). We will consider the case of finite gain \( A \) in parts (f) - (h).

(a) Consider the circuit shown above and \( V_{SS} = -V_{DD} \). What is \( u_+ - u_- \)?

(b) Find \( v_x \) as a function of \( v_{out} \).

(c) What is \( I_{R2} \), i.e. the current flowing through \( R_2 \) as a function of \( v_s \)? **Hint:** Find the current through \( R_1 \) first.

(d) Find \( v_{out} \) as a function of \( v_s \).

(e) What is the current \( i_L \) through the load resistor \( R \)? Give your answer in terms of \( v_{out} \).
(f) We will now examine what happens when \( A \) is not \( \infty \). To understand what happens in this case, first draw an equivalent circuit for the first op amp circuit, by replacing the ideal op-amp in the non-inverting amplifier with the op-amp model shown above. Now, using this setup, calculate \( v_{out} \) and \( v_x \) in terms of \( A \), \( v_s \), \( R_1 \), \( R_2 \) and \( R \). Is the magnitude of \( v_x \) larger or smaller than the magnitude of \( v_s \)? Do these values depend on \( R \)? Hint: Note that the first golden rule still applies, i.e. the currents through the input terminals are zero.

(g) Using your solution to the previous part, calculate the limits of \( v_{out} \) and \( v_x \) as \( A \to \infty \). You should get the same answer as in part (d) for \( v_{out} \).

(h) [OPTIONAL, CHALLENGE] Now you want to make a non-inverting amplifier circuit whose gain is nominally \( G_{nom} = \frac{v_{out}}{v_s} = 1 + \frac{R_2}{R_1} = 4 \). However, \( G_{nom} \) can only be achieved only if the op-amp is ideal, i.e, if its internal gain \( A \to \infty \). But, as with most considerations in the physical world, we must account for nonidealities! In reality, because you will be working with an op-amp with finite gain \( A \), your designed circuit gain may come close to but will never quite reach \( G_{nom} \) as a result of the real op-amp’s finite internal gain \( A \).

Suppose you would like your real op-amp circuit to have a maximum error of 1\% (i.e, a minimum circuit gain of 3.96, i.e. \( \frac{v_{out}}{v_s} \geq 3.96 \)). Remember that only if your op-amp were ideal, you would have a nominal circuit gain of \( G_{nom} = \frac{v_{out}}{v_s} = 1 + \frac{R_2}{R_1} = 4 \).

What is the minimum required value of \( A \), called \( A_{min} \), to achieve that specification? Hint: Use your expression of \( v_{out} \) in part (f) to find an expression for \( G_{min} = \frac{v_{out}}{v_s} \) when \( A \not\to \infty \).

8. Op Amp Nodal Analysis

Consider this op amp circuit below. We are interested in analyzing its input-output relationship, finding the Thevenin equivalent of this op amp circuit, and making some observations about the resulting equivalent.
(a) What is the node voltage at \( u_1 \)?
(b) Write a KCL equation at node \( u_2 \).
(c) Write a KCL equation at node \( u_3 \).
(d) Write an expression relating voltages \( u_4 \) and \( u_5 \).
(e) Noting that this circuit is in negative feedback and putting together every expression we have derived in previous parts, find an expression for \( V_{out} \) as a function of \( V_1 \) and \( V_2 \).
(f) Turn off all independent sources (\( V_1 = V_2 = 0V \)). What is the equivalent resistance as seen between terminals \( a \) and \( b \)? This will be your Thevenin resistance, \( R_{Th} \). (Hint: Consider what the voltage at the output of the op amp becomes and use a test source, or replace the op amp with its internal model where it has a dependent source.)

(g) Use what you found in previous parts to draw the Thevenin equivalent.
(h) Practice (Optional): Does the Thevenin resistance depend on all the resistors or a strict subset? What might explain this?

9. Homework Process and Study Group

Who did you work with on this homework? List names and student ID’s. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.