EECS 16A Designing Information Devices and Systems I
Spring 2020 Discussion 6A

1. True or False? (S16 MT Problem)

You only need to write True or False under each subpart.

(a) There exists an invertible $n \times n$ matrix A for which $A^2 = 0$.

(b) If A is an invertible $n \times n$ matrix, then for all vectors $\vec{b} \in \mathbb{R}^n$, the system $A\vec{x} = \vec{b}$ has a unique solution.

(c) If A and B are invertible $n \times n$ matrices, then the product AB is invertible.

(d) The two vectors $v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ form a basis for the subspace $\text{Span}(\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \})$.

(e) A set of n linearly dependent vectors in $\mathbb{R}^n$ can span $\mathbb{R}^n$.

(f) For all matrices A and B, where A is 5×5 and B is 4×4, it is always the case that $\text{Rank}(A) > \text{Rank}(B)$.

2. Pagerank Review

(a) Consider two linked websites A and B with the following relationship that describes how proportions of visitors move from one to the other:

$$\begin{bmatrix} x_A[k+1] \\ x_B[k+1] \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{6} \\ \frac{1}{2} & \frac{5}{6} \end{bmatrix} \begin{bmatrix} x_A[k] \\ x_B[k] \end{bmatrix}$$

Determine if there is a steady state of visitors on each website and if we converge to it.

(b) Give the fractions of traffic there will be on each site in the long run.

3. Eigenvectors (F17 MT Problem) Consider a matrix $\mathbf{A} \in \mathbb{R}^{3\times 3}$ with eigenvalues $\lambda = 1, 2, 3$, and corresponding eigenvectors $\vec{v}_1$, $\vec{v}_2$ and $\vec{v}_3$ respectively. Let the matrix $\mathbf{B} = \mathbf{A}^3 - 6\mathbf{A}^2 + 11\mathbf{A} - 6\mathbf{I}$.

(a) Find $\mathbf{B}\vec{v}$, where $\vec{v}$ is one of the eigenvectors of $\mathbf{A}$.
Hint:

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = (\lambda - 1)(\lambda - 2)(\lambda - 3)$$

(b) Find all the eigenvalues of the matrix $\mathbf{B}$.

(c) Write out the numerical values in the 3×3 matrix $\mathbf{B}$ and justify your answer.

