EECS 16A Designing Information Devices and Systems I
Spring 2020 Discussion 3B

1. Mechanical Inverses

In each part, determine whether the inverse of $\mathbf{A}$ exists. If it exists, find it.

(a) $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$

(b) $\mathbf{A} = \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}$

(c) $\mathbf{A} = \begin{bmatrix} 5 & 5 & 15 \\ 2 & 2 & 4 \\ 1 & 1 & 4 \end{bmatrix}$

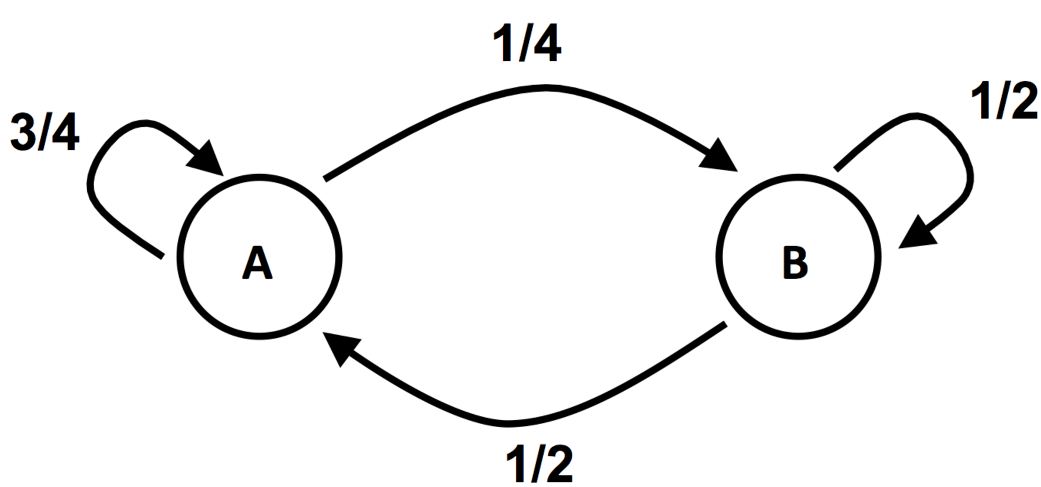
(d) (PRACTICE)

$$\mathbf{A} = \begin{bmatrix} 5 & 5 & 15 \\ 2 & 2 & 4 \\ 1 & 0 & 4 \end{bmatrix}$$

2. Transition Matrix

(a) Suppose there exists some network of pumps as shown in the diagram below. Let $\vec{x}(n) = \begin{bmatrix} x_A(n) \\ x_B(n) \end{bmatrix}$ where $x_A(n)$ and $x_B(n)$ are the states at timestep n.

Find the state transition matrix S, such that $\vec{x}(n+1) = S\vec{x}(n)$.



(b) Let us now find the matrix S^{-1} such that we can recover $\vec{x}(n-1)$ from $\vec{x}(n)$. Specifically, solve for S^{-1} such that $\vec{x}(n-1) = S^{-1}\vec{x}(n)$.

(c) Now draw the state transition diagram that corresponds to the S^{-1} that you just found.

(d) Redraw the diagram from the first part of the problem, but now with the directions of the arrows reversed. Let us call the state transmission matrix of this "reversed" state transition diagram T. Does $T = S^{-1}$?

3. Exploring Column Spaces and Null Spaces

- The **column space** is the **span** of the column vectors of the matrix.
- The **null space** is the set of input vectors that output the zero vector.

For the following matrices, answer the following questions:

i. What is the column space of **A**? What is its dimension?

ii. What is the null space of **A**? What is its dimension?

iii. Are the column spaces of the row reduced matrix **A** and the original matrix **A** the same?

iv. Do the columns of **A** form a basis for $\mathbb{R}^2$? Why or why not?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} -2 & 4 \\ 3 & -6 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & -1 & -2 & -4 \\ 1 & 1 & 3 & -3 \end{bmatrix}$

