EECS 16A Designing Information Devices and Systems I
Spring 2020 Discussion 2A

1. Computations: Inner product and matrix-vector multiplication

(a) For each of the following pairs of vectors, compute their inner product and determine whether they are orthogonal.

i.

$$\vec{a} = \begin{bmatrix} 1 \\ 6 \\ 11 \end{bmatrix} \ \vec{b} = \begin{bmatrix} -6 \\ 1 \\ 2 \end{bmatrix}$$

ii.

$$\vec{a} = \begin{bmatrix} 2 \\ 6 \\ 12 \\ -4 \end{bmatrix} \ \vec{b} = \begin{bmatrix} 6 \\ 2 \\ -1 \\ 3 \end{bmatrix}$$

(b) Perform matrix vector multiplication to compute $A\vec{b}$ in each of the following cases:

i.

$$A = \begin{bmatrix} 1 & 6 \\ 2 & -7 \end{bmatrix} \ \vec{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

ii.

$$A = \begin{bmatrix} 1 & 9 & 2 \\ 7 & 10 & -7 \\ -1 & 2 & -8 \end{bmatrix} \ \vec{b} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

2. Span basics

(a) What is span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$?

(b) Is $\begin{bmatrix} 5 \\ 5 \\ 0 \end{bmatrix}$ in span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$?

(c) What is a possible choice for $\vec{v}$ that would make span$\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \vec{v} \right\} = \mathbb{R}^3$?

(d) For what values of b_1, b_2, b_3 is the following system of linear equations consistent?

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 0 & 0 \end{bmatrix} \vec{x} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

3. Span Proofs

Given some set of vectors $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$, show the following:

(a)

$$\text{span}\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\} = \text{span}\{\alpha\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}, \text{ where } \alpha \text{ is a non-zero scalar}$$

In other words, we can scale our spanning vectors and not change their span.

(b)

$$\text{span}\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\} = \text{span}\{\vec{v}_2, \vec{v}_1, \ldots, \vec{v}_n\}$$

In other words, we can swap the order of our spanning vectors and not change their span.