1. **Linear or Nonlinear**

Determine whether the following functions \( f: \mathbb{R}^2 \to \mathbb{R} \) are linear or nonlinear.

(a) \( f(x_1, x_2) = 3x_1 + 4x_2 \)

(b) \( f(x_1, x_2) = e^{x_2} + x_1^2 \)

(c) \( f(x_1, x_2) = x_2 - x_1 + 3 \)

2. **Solving Systems of Equations**

(a) Systems of linear equations can either have one solution, an infinite number of solutions, or no solution at all. For the following systems of equations, state whether there is a unique solution, no solution, or an infinite number of solutions. If there are an infinite number of solutions give one possible solution.

i. \[
\begin{aligned}
49x + 7y &= 49 \\
42x + 6y &= 42
\end{aligned}
\]

ii. \[
\begin{aligned}
5x + 3y &= -21 \\
2x + y &= -9
\end{aligned}
\]

iii. \[
\begin{aligned}
49x + 7y &= 60 \\
42x + 6y &= 30
\end{aligned}
\]

iv. \[
\begin{aligned}
2x + 2y + 4z &= -1 \\
x + y + z &= -2 \\
x + 2y + 3z &= 2
\end{aligned}
\]

v. \[
\begin{aligned}
2x + 2y + 4z &= 6 \\
y + z &= 1 \\
x + 2y + 3z &= 4
\end{aligned}
\]

vi. \[
\begin{aligned}
x + y + z &= 4 \\
3z &= 6 \\
y + z &= 3
\end{aligned}
\]
(b) Systems of equations can also be interpreted graphically. We will try to build a graphical intuition for the results you found in the previous part. Follow along as your TA walks through dis1B.ipynb.

3. Vectors Introduction to vectors and vector addition.

**Definitions:**

**Vector:** An ordered list of elements - for example:

\[
\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n
\]

\(\mathbb{R}\) or \(\mathbb{R}^1\): The set of all real numbers (i.e. the real line)

\(\mathbb{R}^2\): The set of all two-element vectors with real numbered entries (i.e. plane of \(2 \times 1\) vectors) - for example:

\[
\vec{v} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \in \mathbb{R}^2
\]

\(\mathbb{R}^3\): The set of all three-element vectors with real numbered entries (i.e. 3-space of \(3 \times 1\) vectors) - for example:

\[
\vec{v} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \in \mathbb{R}^3
\]

\(\mathbb{R}^n\): The set of all \(n\)-element vectors with real numbered entries (i.e. \(n\)-space of \(n \times 1\) vectors)

(a) Are the following vectors in \(\mathbb{R}^2\)?

i. \[
\begin{bmatrix} 3 \\ 6 \end{bmatrix}
\]

ii. \[
\begin{bmatrix} 5 \\ 0 \\ 3 \end{bmatrix}
\]

(b) Graphically show the vectors:

i. \[
\begin{bmatrix} 2 \\ 5 \end{bmatrix}
\]

ii. \[
\begin{bmatrix} 5 \\ 2 \end{bmatrix}
\]

(c) Graphically show the vector sum and check your answer algebraically:

\[
\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ 3 \end{bmatrix}
\]