EECS 16A Designing Information Devices and Systems I
Summer 2022 Discussion 0D

1. Solving Systems of Equations

While we'd love every system of linear equations to have a unique solution, in reality we can either have (a) one unique solution, (b) an infinite number of solutions, or (c) no solution at all. We are going to walk through some examples to see what sorts of equations create these types of solutions.

(a) Let's consider the system (where each $a, b \in \mathbb{R}$ can be any real number):

$$\begin{aligned} ax + y &= 3 \\ -x + 2y &= b \end{aligned}$$

For each of the selected values for a and b, sketch or plot out the lines $y(x)$ each of these equations form.

Can you conclude which values result in a unique solution? Infinite solutions? No solutions?

- i. $a = 1$, $b = 0$
- ii. $a = 0$, $b = 2$
- iii. $a = -1/2$, $b = 6$
- iv. $a = -1/2$, $b = 4$

(b) Now, assume we are using the tomography imaging technique described in lecture to image a 2x2 grid, as shown below.

2x2 Tomography Example

x_1	x_2
x_3	x_4

i. We record the following measurements:

$$\begin{aligned} x_1 + x_2 &= 2 \\ x_1 + x_3 &= 2 \\ x_2 + x_4 &= 2 \\ x_3 + x_4 &= 3 \end{aligned}$$

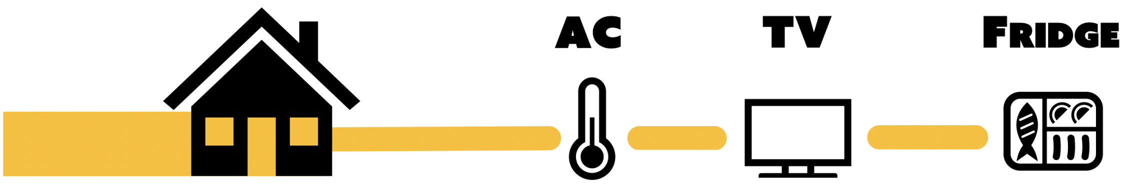
Solve this system using any method. Is this a valid set of measurements? Why or why not?

ii. We are led to believe our measurements might be faulty, so we record the following new measurements:

$$\begin{aligned} x_1 + x_2 &= 2 \\ x_1 + x_3 &= 3 \\ x_2 + x_4 &= 2 \\ x_3 + x_4 &= 3 \end{aligned}$$

Now, solve this system using any method. Is this a valid set of measurements? Why or why not?

2. Energy Disaggregation



Suppose you live in a home with just these **three appliances; an air conditioning unit (AC), a television (TV), and a refrigerator (R)**. Now say you want to find the amount of electricity these appliances use individually, but the only measurement you can take is of the total power your home draws using your meter outside (this is often mounted on the side of the house and shows a running total of your electricity usage).

To do this, you will turn some appliances on and off and then read different total measurements. You can turn off the TV at any time, but you **can't unplug the fridge** since the food would spoil. We keep the air conditioner off throughout the morning, but then it must stay on during the afternoon. However, the breaker trips (meaning the electricity suddenly shuts off) if all three are running, so **the TV and AC cannot run at the same time**.

(a) Can you design a way to calculate how much power each appliance uses? What type of measurements will you need to make, and how many?

Let x_R be the power consumed by the refrigerator, x_{TV} by the TV, and x_{AC} by the AC, and let m_i represent the power measured in measurement i. To find out the values of three variables we somehow need three equations/measurements.

(b) Write out a system of equations that describe your measurements. Can you solve this system so that each appliance's power is written in terms of measurements m_i?

For example: If you measure the power m_1 in the afternoon with the AC and refrigerator on but the TV is off, then the equation might look like $x_{\text{AC}} + x_{\text{R}} = m_1$

(c) Let us say the breaker is fixed, so now we can safely run the AC and TV at the same time. Is there another way (or ways) you could create a new system of equations to solve? If so, see if you can solve your new system!

(d) Lastly suppose, as a busy Berkeley student, you only get a chance to take two measurements. Can you determine how much power each of the three appliances draw? If not, what combinations of power consumption can you find out?