EECS 16A Designing Information Devices and Systems I
Spring 2020 Discussion 8A

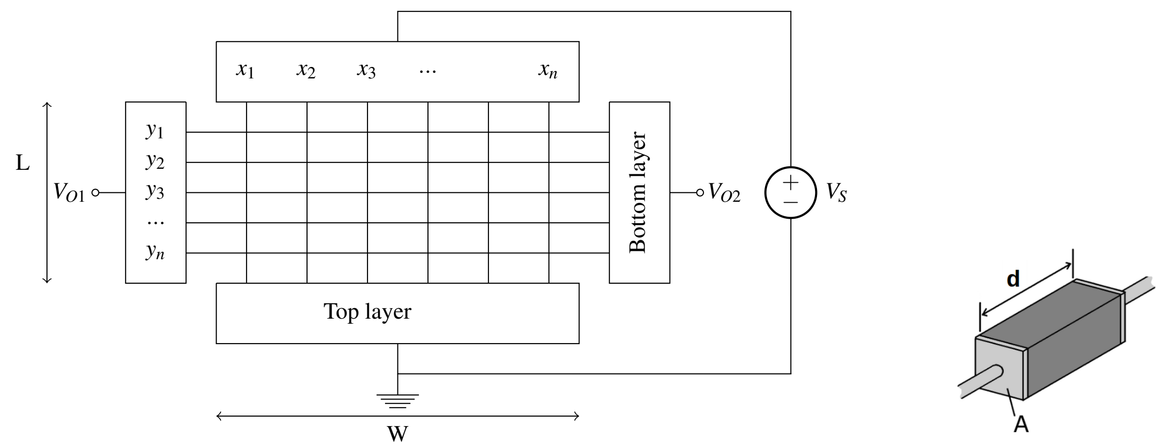
1. Resist the Touch

In this question, we will be re-examining the 2-dimensional resistive touchscreen previously discussed in both lecture and lab. The general touch screen is shown in Figure 1 (a). The touchscreen has length L and width W and is composed of a rigid bottom layer and a flexible upper layer. The strips of a single layer are all connected by an ideal conducting plate on each side. The upper left corner is position $(1,1)$.

The top layer has N vertical strips denoted by $x_1, x_2, \ldots, x_N$. These vertical strips all have cross sectional area A, and resistivity ρ_x.

The bottom layer has N horizontal strips denoted by $y_1, y_2, \ldots, y_N$. These horizontal strips all have cross sectional area A as well, and resistivity ρ_y.

Assume that all top layer resistive strips and bottom layer resistive strips are spaced apart equally. Also assume that all resistive strips are rectangular as shown by Figure 1 (b).



(a) 2-D Resistive Touch Screen

(b) 3D Model of a Single Resistive Strip

Figure 1: Model and components of a general touchscreen

(a) Figure 1(b) shows a model for a single resistive strip. Find the equivalent resistance R_x for the vertical strips and R_y for the horizontal strips, as a function of the screen dimensions W and L, the respective resistivities, and the cross-sectional area A.

Answer: The equation for resistance is $R = \frac{\rho l}{A}$

Therefore, $R_x = \frac{\rho_x L}{A}$.

For the bottom, $R_y = \frac{\rho_y W}{A}$.

(b) Consider a 2×2 example for the touchscreen circuit as in shown in Figure 2.

Given that $V_s = 3\,\text{V}$, $R_x = 2000\,\Omega$, and $R_y = 2000\,\Omega$, draw the equivalent circuit for when the point $(2,2)$ is pressed and solve for the voltage at terminal V_{O2} with respect to ground.

Answer:

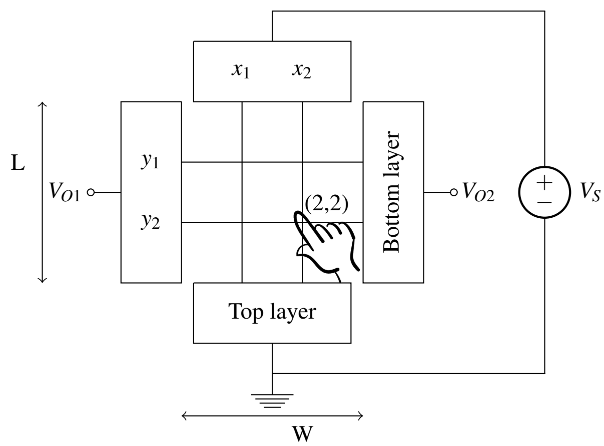
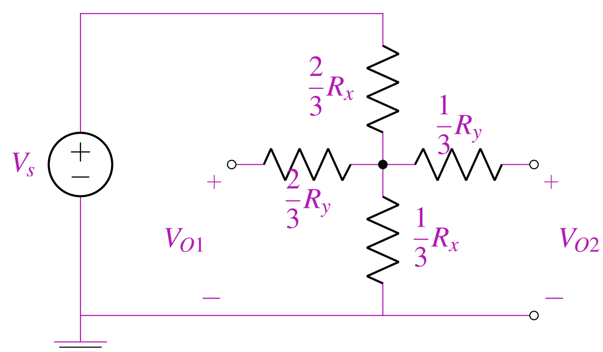


Figure 2: 2×2 Case of the Resistive Touchscreen

Since all of the resistive strips are equally spaced, the resistor above point $(2,2)$ on strip x_2 becomes $\frac{2}{3}R_x$ and the resistor below point $(2,2)$ on strip x_2 becomes $\frac{1}{3}R_x$.

The bottom layer resistors, although they must be drawn in the equivalent circuit, do not affect the voltage at V_{O2} as there are open circuits, leading to no currents through the bottom layer resistors, and therefore no voltage drops over either bottom layer resistor.



Observing that the top layer resistive strips form a voltage divider, we can determine V_{O2} using the voltage divider equation.

Therefore, $V_{O2} = V_{(2,2)} = V_s \dfrac{\frac{1}{3}R_x}{\frac{1}{3}R_x + \frac{2}{3}R_x} = \dfrac{1}{3}V_s = 1\text{V}$.

(c) Suppose a touch occurs at coordinates (i, j) in Figure 1(a). Find an expression for V_{O2} as a function of V_s, N, i, and j. The upper left corner is the coordinate $(1,1)$ and the upper right coordinate is $(N,1)$.

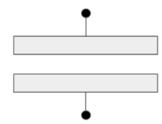
Answer:

$$V_{O2} = \frac{\frac{N+1-j}{N+1}R_x}{R_x}V_s = \frac{N+1-j}{N+1}V_s$$

2. Capacitance Equivalence

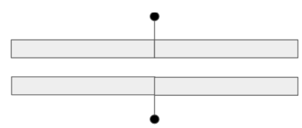
For the structures shown below, assume that the plates have a depth L into the page and a width W and are always a distance d apart.

(a) What is the capacitance of the structure shown below?



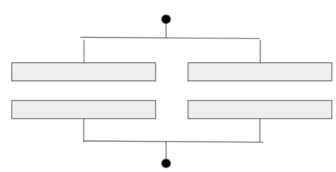
Answer:
The capacitance of two parallel plate conductors is given by $C = \varepsilon \frac{A}{d}$. The cross-sectional area A is WL, so the capacitance is $C = \varepsilon \frac{WL}{d}$.

(b) Suppose that we take two such structures and put them next to each other as shown below. What is the capacitance of this new structure?



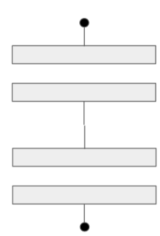
Answer:
Here, we have just doubled the width of the capacitor plates. The new capacitance is $C = \varepsilon \frac{2WL}{d}$. Notice that this is just double the capacitance from the first part.

(c) Now suppose that rather than connecting the together as shown above, we connect them with an ideal wire as shown below. What is the capacitance of this structure?



Answer:
Intuitively, nothing has changed here since we have just added an ideal wire between two capacitors. Thus, the answer remains $C = \varepsilon \frac{2WL}{d}$.

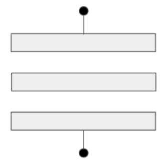
(d) Suppose that we now take two capacitors and connect them as shown below. What is the capacitance of the structure?



Answer:
We know that capacitors placed in series follow the parallel rule. Thus, the overall capacitance is half the individual capacitance.

$$C_{eq} = C||C = \frac{C \cdot C}{C+C} = \frac{C}{2}$$

(e) What is the capacitance of the structure shown below?



Answer:

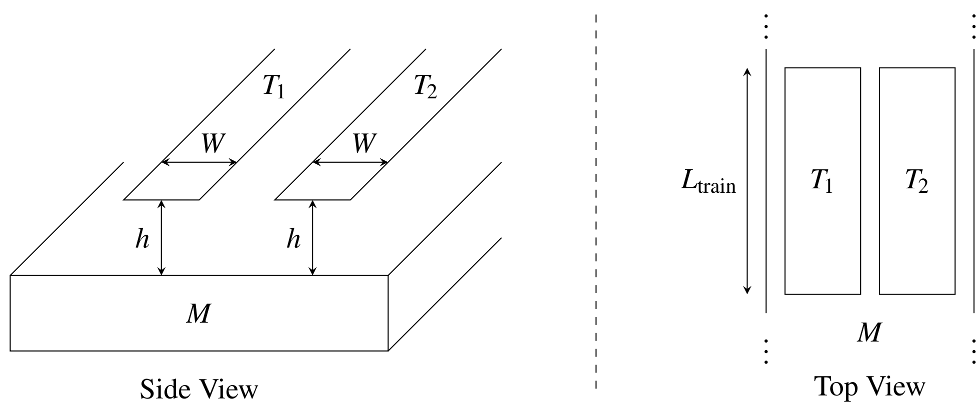
Notice here that we are ignoring the material in the middle. Thus, from a modeling perspective, we can think of this as the original capacitor with the distance between the plates doubled.

$$C_{eq} = \varepsilon \frac{WL}{2d} = \frac{1}{2} \varepsilon \frac{WL}{2d} = \frac{C}{2}$$

3. Maglev Train Height Control System

One of the fastest forms of land transportation are trains that actually travel slightly elevated from ground using magnetic levitation (or "maglev" for short). Ensuring that the train stays at a relatively constant height above its "tracks" (the tracks in this case are what provide the force to levitate the train and propel it forward) is critical to both the safety and fuel efficiency of the train. In this problem, we'll explore how the maglev trains use capacitors to keep them elevated. (Note that real maglev trains may use completely different and much more sophisticated techniques to perform this function, so if you e.g. get a contract to build such a train, you'll probably want to do more research on the subject.)

(a) As shown below, let's imagine that all along the bottom of the train, we put two parallel strips of metal (T_1, T_2), and that on the ground below the train (perhaps as part of the track), we have one solid piece of metal (M).



Assuming that the entire train is at a uniform height above the track and ignoring any fringing fields (i.e., all capacitors are purely parallel plate), as a function of L_{train} (the length of the train), W (the width of T_1/T_2), and h (the height of the train off of the track), what is the capacitance between T_1 and M? How about the capacitance between T_2 and M?

Answer:

The distance between the plates (T_1 & M or T_2 & M) is h. The area of plate for the parallel plate capacitor is $A = WL_{\text{train}}$. Using the formula for capacitance of a parallel plate capacitor, we get:

$$C = \frac{\varepsilon A}{d}$$

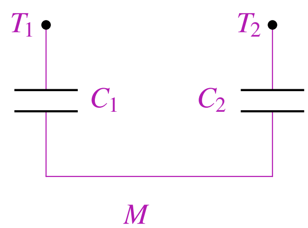
$$C_1 = \frac{\varepsilon W L_{\text{train}}}{h} \text{ (Capacitance between T}_1\text{ and M)}$$

$$C_2 = \frac{\varepsilon W L_{\text{train}}}{h} \text{ (Capacitance between T}_2\text{ and M)}$$

(b) Any circuit on the train can only make direct contact at T_1 and T_2. To detect the height of the train, it would only be able to measure the effective capacitance between T_1 and T_2. Draw a circuit model showing how the capacitors between T_1 and M and between T_2 and M are connected to each other.

Answer:

The capacitors C_1 and C_2 are in series. To realize this, let's consider the train circuit that is in contact with T_1 and T_2. If there is current entering plate T_1, the same current has to exit plate T_2. Thus, the circuit can be modeled as follows:



(c) Using the same parameters as in part (a), provide an expression for the capacitance between T_1 and T_2.

Answer:

Since the two capacitors are in series, the effective capacitance between T_1 and T_2 is given by:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

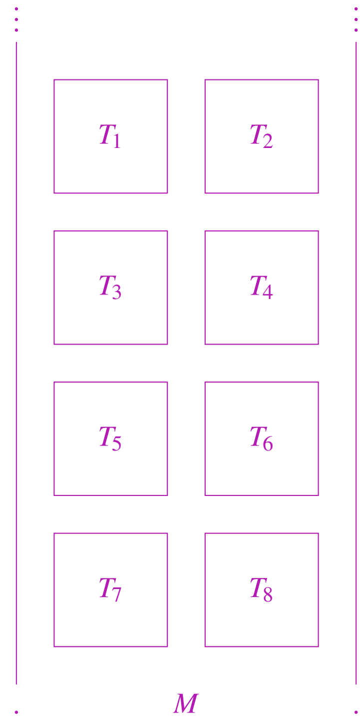
Thus, we get

$$\frac{1}{C_{\text{eq}}} = \frac{h}{\varepsilon W L_{\text{train}}} + \frac{h}{\varepsilon W L_{\text{train}}}$$

$$C_{\text{eq}} = \frac{\varepsilon W L_{\text{train}}}{2h}$$

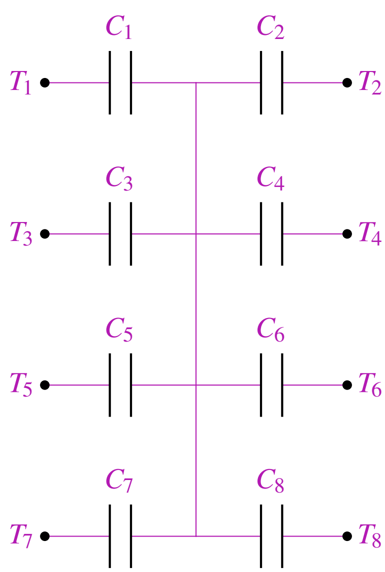
(d) So far we've assumed that the height of the train off of the track is uniform along its entire length, but in practice, this may not be the case. Suggest and sketch a modification to the basic sensor design (i.e., the two strips of metal T_1/T_2 along the entire bottom of the train) that would allow you to measure the height at the train at 4 different locations.

Answer:



Top View

One important thing to note about this circuit is that it works only if extra care is taken during the capacitance measurement circuit. The equivalent model for this is:



Therefore, the circuit needs separate switches on each T, so that you can measure the capacitance between only two terminals (like T_1 & T_2) and so that the effect of other capacitors is nullified.