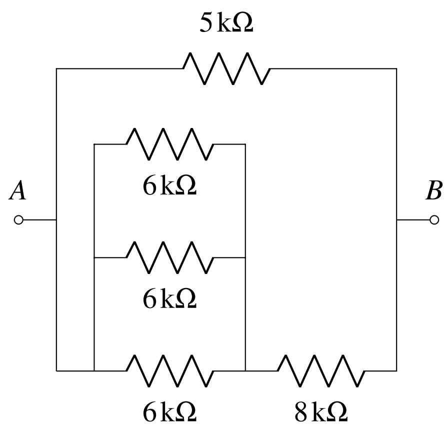
EECS 16A Designing Information Devices and Systems I
Spring 2020 Discussion 7B

1. Series and Parallel Combinations

For the resistor network shown below, find an equivalent resistance between the terminals A and B using the resistor combination rules for series and parallel resistors.



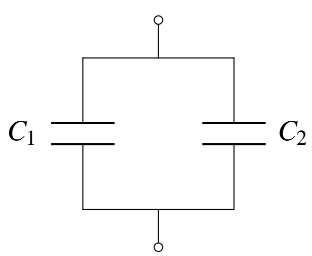
Answer:

$$5\,\mathrm{k}\Omega \parallel ((6\,\mathrm{k}\Omega \parallel 6\,\mathrm{k}\Omega \parallel 6\,\mathrm{k}\Omega) + 8\,\mathrm{k}\Omega) = 5\,\mathrm{k}\Omega \parallel (2\,\mathrm{k}\Omega + 8\,\mathrm{k}\Omega) = 5\,\mathrm{k}\Omega \parallel 10\,\mathrm{k}\Omega = 3.33\,\mathrm{k}\Omega$$

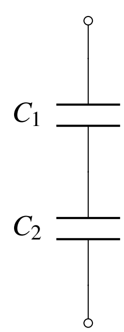
2. Series And Parallel Capacitors

Derive C_{eq} for the following circuits.

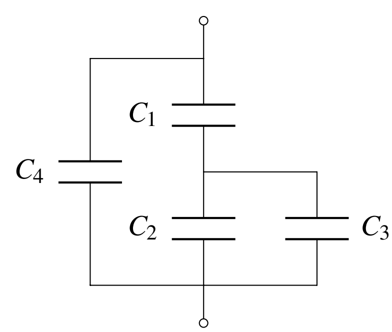
(a)



(b)



(c)

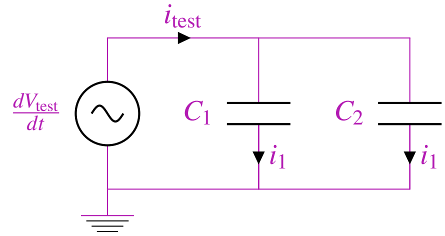


Answer:

(a)

$$C_{eq} = C_1 + C_2$$

Notice these capacitors are in parallel. We can derive their equivalent capacitance by connecting them to a voltage source with a constant derivative, as shown by the circuit below:



Since both capacitors have the same voltage across them:

$$\frac{dV_{C_1}}{dt} = \frac{dV_{C_2}}{dt} = \frac{dV_{\text{test}}}{dt}$$

$$i_1 = C_1 \frac{dV_{\text{test}}}{dt}$$

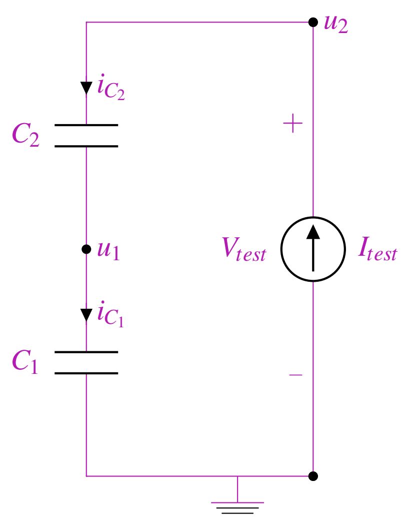
$$i_2 = C_2 \frac{dV_{\text{test}}}{dt}$$

$$i_t = i_1 + i_2 = (C_1 + C_2)\frac{dV_{\text{test}}}{dt}$$

Since we know $i_{\text{test}} = C_{\text{eq}} \frac{dV_{\text{out}}}{dt}$,

$$C_{\text{eq}} = C_1 + C_2$$

(b) In order to find the equivalence capacitance of the circuit, we plug in a test current source, and measure the rate of change of voltage across it.



From KCL, we know that all of the currents are equal.

$$i_{C_1} = i_{C_2} = I_{test}$$

For each capacitor, we plug in our $I - \frac{dV}{dt}$ relationship:

$$i_{C_1} = I_{test} = C_1 \frac{du_1}{dt}$$

$$i_{C_2} = I_{test} = C_2 \frac{d(u_2 - u_1)}{dt} = C_2 \left(\frac{du_2}{dt} - \frac{du_1}{dt} \right)$$

Next, we eliminate u_1 from the equations above and rearrange.

$$\frac{du_1}{dt} = \frac{I_{test}}{C_1} \Rightarrow I_{test} = C_2 \frac{du_2}{dt} - \frac{C_2}{C_1} I_{test}$$

$$I_{test} = \frac{C_2}{1 + \frac{C_2}{C_1}} \frac{du_2}{dt}$$

Finally, we plug in that $u_2 = V_{test}$ and solve for the equivalent capacitance with $C_{eq} = I_{test} / \frac{dV_{test}}{dt}$

$$I_{test} = \frac{C_2}{1 + \frac{C_2}{C_1}} \frac{dV_{test}}{dt}$$

$$\Rightarrow C_{eq} = \frac{C_2}{1 + \frac{C_2}{C_1}} = \frac{C_1 C_2}{C_1 + C_2}$$

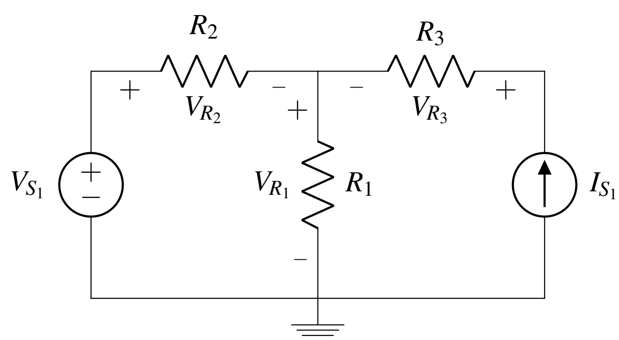
Note that this is the same as saying $C_{eq} = C_1 \parallel C_2$. Remember that the $\parallel$ operator is mathematical notation; in this case, the capacitors are actually in series, but *mathematically* their equivalent circuit is found via the "parallel resistor" operation.

(c) Given that we know what the relationship for capacitors in series and parallel are from the last two parts, we can just simply the capacitors step by step:

$$C_{eq} = (C_4 + (C_1 \parallel (C_2 + C_3))) = \frac{C_4(C_1 + C_2 + C_3) + C_1(C_2 + C_3)}{C_1 + C_2 + C_3}$$

3. Superposition

For the following circuit, use the superposition theorem to solve for the voltages across the resistors.



Answer: Turning on only V_{S_1}, we have the following voltages across the resistors:

$$V_{R_1} = \frac{R_1}{R_1 + R_2} V_{S_1}$$

$$V_{R_2} = \frac{R_2}{R_1 + R_2} V_{S_1}$$

$$V_{R_3} = 0$$

Then turning only I_{S_1}, we have the following voltages:

$$V_{R_1} = \frac{R_1 R_2}{R_1 + R_2} I_{S_1}$$
$$V_{R_2} = -\frac{R_1 R_2}{R_1 + R_2} I_{S_1}$$
$$V_{R_3} = I_{S_1} R_3$$

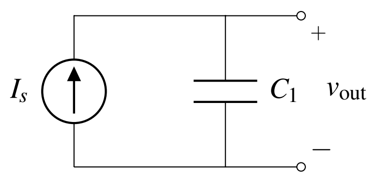
Using superposition we can sum up the contributions from both V_{S_1} and I_{S_1} to get:

$$V_{R_1} = \frac{R_1}{R_1 + R_2} V_{S_1} + \frac{R_1 R_2}{R_1 + R_2} I_{S_1}$$
$$V_{R_2} = V_{S_1} - V_{R_1} = \frac{R_2}{R_1 + R_2} V_{S_1} - \frac{R_1 R_2}{R_1 + R_2} I_{S_1}$$
$$V_{R_3} = I_{S_1} R_3$$

4. Current Sources And Capacitors

For the circuits given below, give an expression for $v_{\text{out}}(t)$ in terms of I_s, C_1, C_2, and t. Assume that all capacitors are initially uncharged, i.e. the initial voltage across each capacitor is 0 V.

(a)

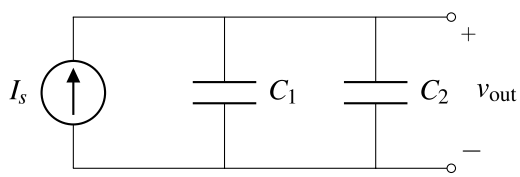


Answer:

$$I_s = C_1 \frac{dv_{\text{out}}(t)}{dt}$$
$$v_{\text{out}}(t) = \int \frac{I_s}{C_1} dt = \frac{I_s t}{C_1} + v_{\text{out}}(0)$$

Since the capacitor is initially uncharged, $v_{\text{out}}(0) = 0$, so $v_{\text{out}}(t) = \frac{I_s t}{C_1}$.

(b)



Answer:

We can combine the two capacitors into an equivalent capacitor with capacitance $C_1 + C_2$. Again, $v_{\text{out}}(0) = 0$ because all capacitors are initially uncharged.

$$I_s = (C_1 + C_2) \frac{dv_{\text{out}}(t)}{dt}$$
$$v_{\text{out}}(t) = \frac{I_s t}{C_1 + C_2} + v_{\text{out}}(0) = \frac{I_s t}{C_1 + C_2}$$

