1. Nodes and Branches

In the circuit shown below, label and count all nodes and branches.

Answer: There are seven nodes and nine branches.
2. Divider

For the circuit below, find the voltage $V_{out}$ in terms of the resistances $R_1$, $R_2$, and $V_s$.

![Diagram of the circuit](image)

**Answer:**

In this example we will go through all the steps that we saw in lecture one-by-one.

Step 1: Select a ground node,

![Diagram showing the selection of a ground node](image)

Step 2: Label all nodes with voltage set by voltage sources (denoted below as $u_1$),

![Diagram showing the labeling of nodes](image)
Step 3: Label remaining nodes (denoted below as $u_2$).

Step 4: Label element voltages and currents,

Step 5: Write KCL equations for all nodes with unknown voltages (namely $u_2$):

$$I_{R_2} = I_{R_1}$$

Step 6: Find expressions for all element currents in terms of element voltages and characteristics,

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{u_1 - u_2}{R_1} = \frac{V_s - u_2}{R_1}$$

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{u_2 - 0}{R_2}$$
Where we used the fact that \( u_1 = V_s \)

Step 7: Substitute expressions found in 6 into the KCL equations from step 5,

\[
IR_2 = IR_1 \\
\Rightarrow \frac{V_s - u_2}{R_1} = \frac{u_2 - 0}{R_2} \\
\Rightarrow (V_s - u_2)R_2 = u_2R_1 \\
\Rightarrow u_2 = \frac{R_2}{R_1 + R_2}V_s
\]

Now we have two unknowns, \( i_s \) and \( u_2 \), and two equations. We can solve them directly for \( u_2 \). Notice that \( V_{out} = u_2 - 0 = u_2 \)

\[
V_{out} = u_2 = \frac{R_2}{R_1 + R_2}V_s
\]

3. A Simple Circuit

Use KVL and/or KCL to solve the following circuits.

(a) For this problem assume \( V_S = 1V \) and \( R_1 = 1k\Omega \). Find the current, \( I_s \) flowing through the voltage source.

\[
\text{Answer: } \text{Since this circuit has only 2 terminals we can find the current flowing through } I_s \text{ without precisely following the algorithm outlined in lecture. In fact we can even not assign a ground potential. Labeling element voltages and currents we have:}
\]

\[
\begin{align*}
V_S & \quad I_s \\
R_1 & \quad - I_R_2 \\
\end{align*}
\]

Using KVL and Ohm’s law we get:

\[
\begin{align*}
V_S &= V_{R_1} \\
I_{R_1} &= \frac{V_{R_1}}{R_1} = \frac{V_S}{R_1} \
\end{align*}
\] (1)

\[
\begin{align*}
I_s + I_{R_1} &= 0 \Rightarrow I_s = -I_{R_1} = -\frac{V_S}{R_1} = -1mA \
\end{align*}
\] (3)

(b) For this problem assume \( V_S = 1V, R_1 = 2k\Omega, \) and \( R_2 = 2k\Omega \). Find the current, \( I_s \) flowing through the voltage source.
Answer: Again we can follow the same procedure since we still have two terminals. Let’s label again all element voltages and currents.

Here we have also assigned a ground node and labeled the other node of the circuit as $u_1$. Once again we will use KVL and Ohm’s law:

\[ V_S = V_{R_1} = V_{R_2} \]  
\[ I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_S}{R_1} \]  
\[ I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_S}{R_2} \]

Then writing out KCL and substituting from above we have:

\[ I_{R_1} + I_{R_2} + I_S = 0 \]  
\[ \frac{V_S}{R_1} + \frac{V_S}{R_2} + I_S = 0 \Rightarrow I_S = -\left( \frac{V_S}{R_1} + \frac{V_S}{R_2} \right) \]

Plugging in,

\[ I_S = -1mA \]

Notice that we did not make use of node $u_1$ or the ground node anywhere.

(c) Now, instead of a voltage source, we have a current source ($I_s$) in our circuit. Find the currents $I_1$ and $I_2$ flowing through each of the resistors in terms of $R_1, R_2, I_s$.

Answer: Let $V$ denote the voltage drop across the resistors. By KVL, we realize that the voltage drop across each of the resistors is the same. Using KCL, we realize that $I_s = I_1 + I_2$. Using Ohm’s law, we can write the above as $I_s = \frac{V}{R_1} + \frac{V}{R_2} = \frac{V_R + R_1}{R_1 + R_2}$. Hence, $I_1 = \frac{V}{R_1} = \frac{I_s R_2}{R_1 + R_2}$ and $I_2 = \frac{V}{R_2} = \frac{I_s R_1}{R_1 + R_2}$.