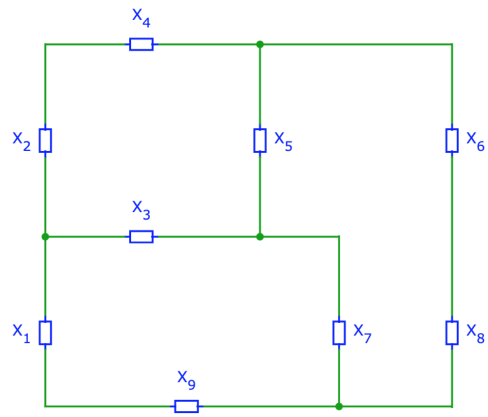
EECS 16A Designing Information Devices and Systems I
Spring 2020 Discussion 6B

1. Nodes and Branches

In the circuit shown below, label and count all nodes and branches.



Answer: There are seven nodes and nine branches.

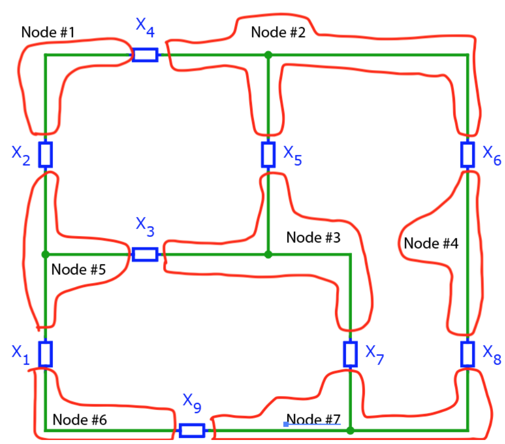


Figure 1: Labeled Nodes

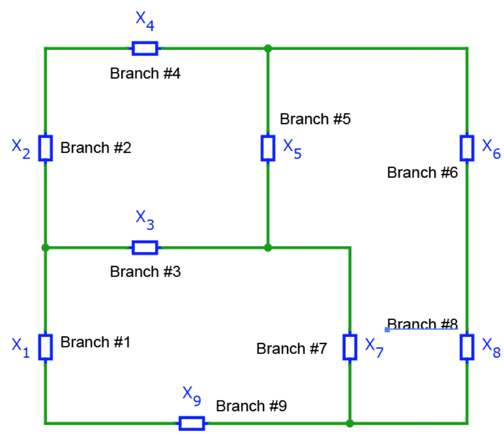
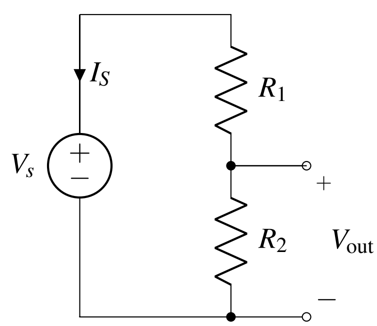


Figure 2: Labeled branches

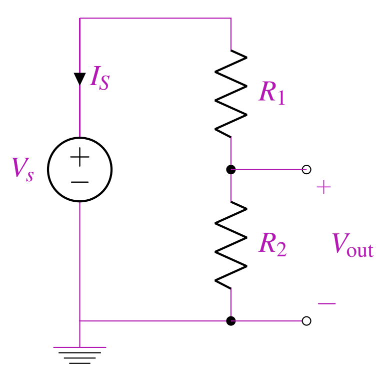
2. Divider

For the circuit below, find the voltage V_{out} in terms of the resistances R_1, R_2, and V_s.

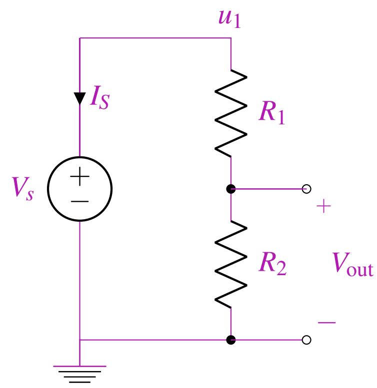


Answer:

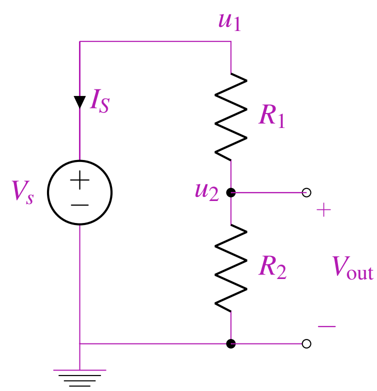
In this example we will go through all the steps that we saw in lecture one-by-one.
Step 1: Select a ground node,



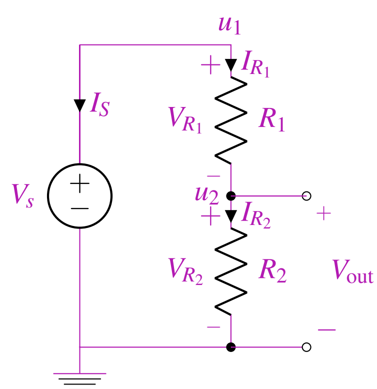
Step 2: Label all nodes with voltage set by voltage sources (denoted below as u_1),



Step 3: Label remaining nodes (denoted below as u_2),



Step 4: Label element voltages and currents,



Step 5: Write KCL equations for all nodes with unknown voltages (namely u_2):

$$I_{R_2} = I_{R_1}$$

Step 6: Find expressions for all element currents in terms of element voltages and characteristics,

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{u_1 - u_2}{R_1} = \frac{V_s - u_2}{R_1}$$
$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{u_2 - 0}{R_2}$$

Where we used the fact that $u_1 = V_s$

Step 7: Substitute expressions found in 6 into the KCL equations from step 5,

$$\begin{aligned} I_{R_2} &= I_{R_1} \\ &\Rightarrow \frac{V_s - u_2}{R_1} = \frac{u_2 - 0}{R_2} \\ &\Rightarrow (V_s - u_2)R_2 = u_2 R_1 \\ &\Rightarrow u_2 = \frac{R_2}{R_1 + R_2} V_s \end{aligned}$$

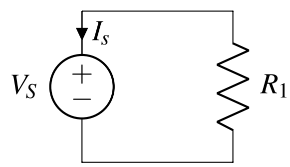
Now we have two unknowns, i_x and u_2, and two equations. We can solve them directly for u_2. Notice that $V_{\text{out}} = u_2 - 0 = u_2$

$$V_{\text{out}} = u_2 = \frac{R_2}{R_1 + R_2} V_s$$

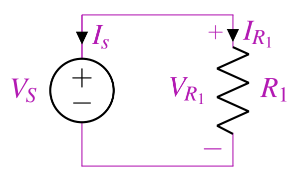
3. A Simple Circuit

Use KVL and/or KCL to solve the following circuits.

(a) For this problem assume $V_S = 1V$ and $R_1 = 1k\Omega$. Find the current, I_s flowing through the voltage source.



Answer: Since this circuit has only 2 terminals we can find the current flowing through I_s without precisely following the algorithm outlined in lecture. In fact we can even not assign a ground potential. Labeling element voltages and currents we have:



Using KVL and Ohm's law we get:

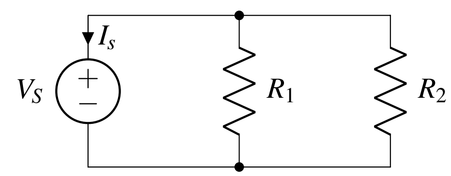
$$V_S = V_{R_1} \tag{1}$$

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_S}{R_1} \tag{2}$$

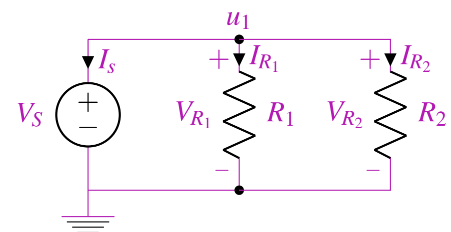
Finally through KCL on the top node we obtain:

$$I_s + I_{R_1} = 0 \Rightarrow I_s = -I_{R_1} = -\frac{V_S}{R_1} = -1mA \tag{3}$$

(b) For this problem assume $V_S = 1V$, $R_1 = 2k\Omega$, and $R_2 = 2k\Omega$. Find the current, I_s flowing through the voltage source.



Answer: Again we can follow the same procedure since we still have two terminals. Let's label again all element voltages and currents.



Here we have also assigned a ground node and labeled the other node of the circuit as u_1. Once again we will use KVL and Ohm's law:

$$V_S = V_{R_1} = V_{R_2} \tag{4}$$

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_S}{R_1} \tag{5}$$

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_S}{R_2} \tag{6}$$

$$\tag{7}$$

Then writing out KCL and substituting from above we have:

$$I_{R_1} + I_{R_2} + I_S = 0 \tag{8}$$

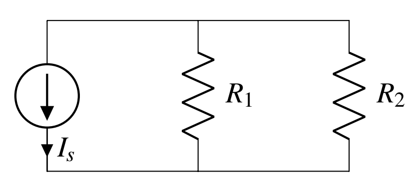
$$\frac{V_s}{R_1} + \frac{V_s}{R_2} + I_S = 0 \Rightarrow I_S = -\left(\frac{V_s}{R_1} + \frac{V_s}{R_2}\right) \tag{9}$$

Plugging in,

$$I_S = -1mA \tag{10}$$

Notice that we did not make use of node u_1 or the ground node anywhere.

(c) Now, instead of a voltage source, we have a current source (I_s) in our circuit. Find the currents I_1 and I_2 flowing through each of the resistors in terms of R_1, R_2, I_s.



Answer: Let V denote the voltage drop across the resistors. By KVL, we realize that the voltage drop across each of the resistors is the same. Using KCL, we realize that $I_s = I_1 + I_2$. Using Ohm's law, we can write the above as $I_s = \frac{V}{R_1} + \frac{V}{R_2} = V\frac{R_2+R_1}{R_1R_2}$. Hence, $I_1 = \frac{V}{R_1} = \frac{I_sR_2}{R_1+R_2}$ and $I_2 = \frac{V}{R_2} = \frac{I_sR_1}{R_1+R_2}$