EECS 16A Designing Information Devices and Systems I
Spring 2020 Discussion 1A

1. Systems of Equations

Solve the following systems of equations, or if there is no solution, explain why. Plot parts (a) and (b).

(a)

$$\begin{cases} 2x + y = 6 \\ 3x - 2y = 2 \end{cases}$$

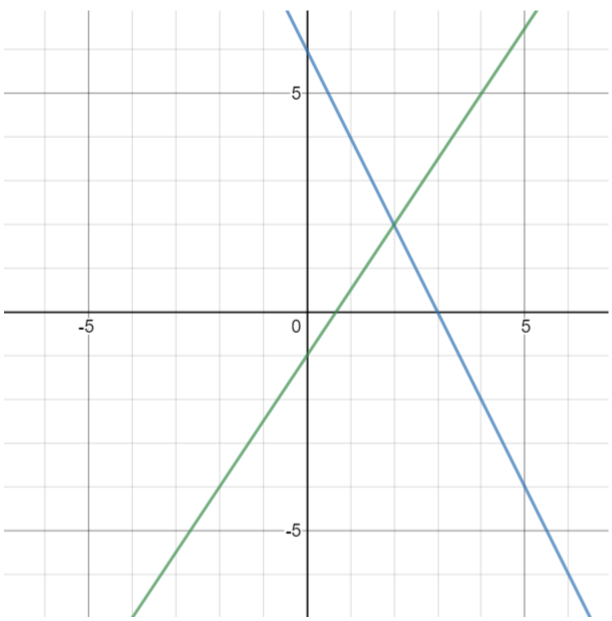
Answer:

There are many ways to solve systems of linear equations, here we will use substitution.

$$\begin{aligned} 2x + y = 6 &\implies y = 6 - 2x \\ 3x - 2(6 - 2x) &= 2 \\ 7x &= 14 \\ x &= 2 \\ y &= 6 - 2(2) = 2 \end{aligned}$$

To plot let's write each linear equation in the form of $y = mx + b$.
As a reminder m is the slope and b is the y-intercept.

$$\begin{aligned} 2x + y = 6 &\implies y = -2x + 6 \\ 3x - 2y = 2 &\implies y = \frac{3}{2}x - 1 \end{aligned}$$



(b)

$$\begin{cases} 6x+2y=15 \\ 3x+y=7 \end{cases}$$

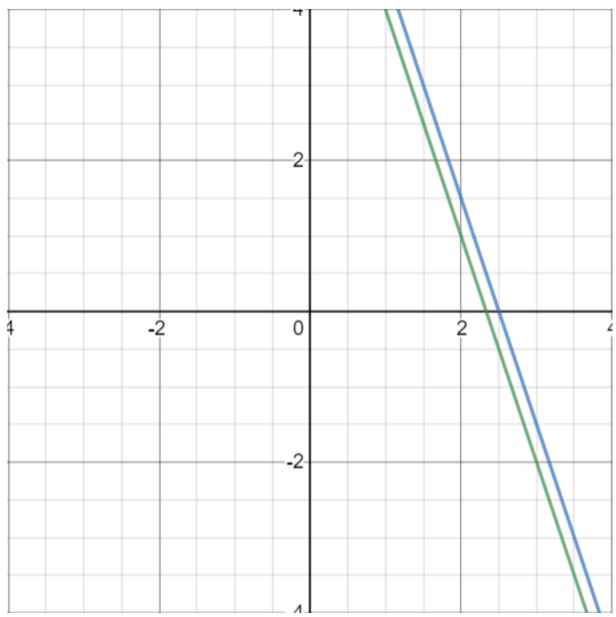
Answer:

Notice that if you multiply the second equation by 2, you obtain $6x+2y=14$. This is inconsistent with the first equation, as $6x+2y=15$, therefore there is no solution.

To plot let's write each linear equation in the form of $y=mx+b$.

As a reminder m is the slope and b is the y-intercept.

$$6x+2y=15 \implies y=-3x+\frac{15}{2}$$
$$3x+y=7 \implies y=-3x+7$$



(c)

$$\begin{cases} x+y+z=2 \\ x-y=1 \\ 2y+z=1 \end{cases}$$

Answer:

To solve this system of linear equations, we will begin by subtracting the second equation from the first equation.

$$2y+z=1$$

Notice that this equation is the same as equation 3. Therefore, the system of linear equations does not have a unique solution, infact it has infinitely many solutions.

The set of solutions can be described by a set of parameteric equations. To find the equations, we begin by chosing a parameter t, and set one of the variables equal to t, we chose z. Then we can write the other variables in terms of z and thus t.

$$\begin{aligned}
z &= t \\
y &= \frac{1-z}{2} = \frac{1-t}{2} \\
x &= 2 - y - z = 2 - \frac{1-t}{2} - t = \frac{3}{2} - \frac{1}{2}t
\end{aligned}$$

2. Finding The Bright Cave

Nara the one-handed druid and Kody the one-handed ranger find themselves in dire straits. Before them is a cliff with four cave entrances arranged in a square: two upper caves and two lower caves. Each entrance emits a certain amount of light, and the two wish to find exactly the amount of light coming from each cave. Here's the catch: after contracting a particularly potent strain of ghoul fever, our intrepid heroes are only able to see the total intensity of light before them (so their eyes operate like a single-pixel camera). Kody and Nara are capable adventurers, but they don't know any linear algebra – and they need your help.

Kody proposes an imaging strategy where he uses his hand to completely block the light from two caves at a time. He is able to take measurements using the following four masks (black means the light is blocked from that cave):

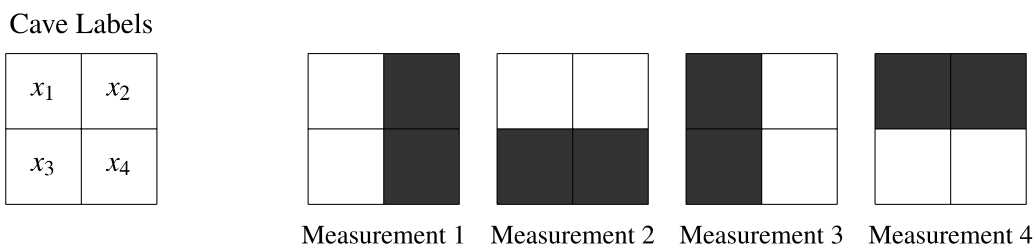


Figure 1: Four image masks.

(a) Let x_1, x_2, x_3, and x_4 represent the magnitude of light emanating from the four cave entrances shown in the image above. Write an equation for each masking process in Figure 1 which results in the four measurements of total light: m_1, m_2, m_3, and m_4.

Answer:

$$m_1 = x_1 + x_3$$
$$m_2 = x_1 + x_2$$
$$m_3 = x_2 + x_4$$
$$m_4 = x_3 + x_4$$

(b) Does Kody's set of masks give us a unique solution for all four caves' light intensities? Why or why not?

Answer:

Notice the equations. If we find that we could get one equation from the other equations, then we know that the solution is not unique. Notice that the sum of the first and the third row is the same is the sum of the second and fourth row.

$$\begin{aligned} m_1 + m_3 &= m_2 + m_4 \\ m_4 &= m_1 + m_3 - m_2 \\ (x_3 + x_4) &= (x_1 + x_3) + (x_2 + x_4) - (x_1 + x_2) \\ x_3 + x_4 &= x_3 + x_4 \end{aligned}$$

(c) Nara, in her infinite wisdom, places her one hand diagonally across the entrances, covering two of the cave entrances. However, her hand is not wide enough, letting in 50% of the light from the caves covered and 100% of the light from the caves not covered. The following diagram shows the percentage of light let through from each cave:

50%	100%
100%	50%

Does this additional measurement give them enough information to solve the problem? Why or why not?

Answer:

The answer is yes; the additional measurement does give them enough information to solve the problem. Since Nara's measurement is linearly independent from the other four, we are now able to solve for all four light intensities uniquely.

This can be shown using algebra with the addition of the following equation:

$$m_5 = \frac{1}{2}x_1 + x_2 + x_3 + \frac{1}{2}x_4$$

Note that we can isolate x_3 by combining measurements 2, 3, and 5:

$$x_3 = m_5 - \frac{1}{2}m_2 - \frac{1}{2}m_3$$

We can use further substitution to determine x_1, x_3, and x_4:

$$x_1 = m_1 - x_3 = m_1 - (m_5 - \frac{1}{2}m_2 - \frac{1}{2}m_3) = m_1 - m_5 + \frac{1}{2}m_2 + \frac{1}{2}m_3$$
$$x_2 = m_2 - x_1 = m_2 - (m_1 - m_5 + \frac{1}{2}m_2 + \frac{1}{2}m_3) = -m_1 + m_5 + \frac{1}{2}m_2 - \frac{1}{2}m_3$$
$$x_4 = m_4 - x_3 = m_4 - (m_5 - \frac{1}{2}m_2 - \frac{1}{2}m_3) = m_4 - m_5 + \frac{1}{2}m_2 + \frac{1}{2}m_3$$