1. Correlation

(a) You are given the following two signals:

\[
\begin{align*}
\text{Signal 1} & : 4 \quad -2 \quad 0 \quad 0 \quad -2 \quad 0 \quad 0 \quad 0 \\
\text{Signal 2} & : -4 \quad 8 \quad -4 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0
\end{align*}
\]

Sketch the linear cross-correlation of signal 1 with signal 2, that is find: \(\text{corr}(\vec{s}_1, \vec{s}_2)[n]\) for \(n = 0, 1, \ldots, 4\). Do not assume the signals are periodic.

**Answer:**

Represent signal 1 as the vector \(\vec{s}_1 = [4 \quad -2 \quad 0 \quad 0 \quad -2 \quad 0 \quad 0 \quad 0 \quad 0]^T\), zero-padded so that we compute only the linear correlation. Similarly, represent signal 2 as the vector \(\vec{s}_2 = [-4 \quad 8 \quad -4 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]\), where we once again zero pad the vector. Notice that we zero pad the vectors \(\vec{s}_1\) and \(\vec{s}_2\) to represent the signals from \(n = 0, 1, \ldots, 8\). This is because we are only interested in calculating the cross-correlation for \(n = 0, 1, \ldots, 4\), therefore we will only need to shift the vector \(\vec{s}_2\) four times.

The cross-correlation between two vectors is defined as follows:

\[
\text{corr}(\vec{x}, \vec{y})[k] = \sum_{i=-\infty}^{\infty} \vec{x}[i] \vec{y}[i-k]
\]

To compute the cross-correlation \(\text{corr}(\vec{s}_1, \vec{s}_2)[n]\), we shift the vector \(\vec{s}_2\) and compute the inner product of the shifted \(\vec{s}_2\) and the vector \(\vec{s}_1\).

\[
\begin{align*}
\vec{s}_1 & : 4 \quad -2 \quad 0 \quad 0 \quad -2 \quad 0 \quad 0 \quad 0 \\
\vec{s}_2[n] & : -4 \quad 8 \quad -4 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
\langle\vec{s}_1, \vec{s}_2[n]\rangle & : -16 \quad -16 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad = -32 \\
\vec{s}_1 & : 4 \quad -2 \quad 0 \quad 0 \quad -2 \quad 0 \quad 0 \quad 0 \\
\vec{s}_2[n-1] & : 0 \quad -4 \quad -8 \quad -4 \quad 0 \quad 0 \quad 0 \quad 0 \\
\langle\vec{s}_1, \vec{s}_2[n-1]\rangle & : 0 \quad 8 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad = 8 \\
\vec{s}_1 & : 4 \quad -2 \quad 0 \quad 0 \quad -2 \quad 0 \quad 0 \quad 0 \\
\vec{s}_2[n-2] & : 0 \quad 0 \quad -4 \quad 8 \quad -4 \quad 0 \quad 0 \quad 0 \\
\langle\vec{s}_1, \vec{s}_2[n-2]\rangle & : 0 \quad 0 \quad 0 \quad 0 \quad 8 \quad 0 \quad 0 \quad 0 \quad 0 \quad = 8
\end{align*}
\]
Non-periodic Cross-correlation of Signals 1 and 2

(b) Now the pattern in $\tilde{s}_1$ is repeated three times:

Sketch the linear cross-correlation of signal 1 with signal 2, $\text{corr}(\tilde{s}_1, \tilde{s}_2)[n]$, for $n = 0, 1, \ldots, 4$.

**Answer:** Recall that $\text{corr}(\tilde{x}, \tilde{y})[k] = \sum_{i=-\infty}^{\infty} \tilde{x}[i] \tilde{y}[i-k]$.

As we did in part a) to compute the cross-correlation $\text{corr}(\tilde{s}_1, \tilde{s}_2)$, we shift the vector $\tilde{s}_2$ and compute the inner product of the shifted $\tilde{s}_2$ and the vector $\tilde{s}_1$. Since we are interested in $\text{corr}(\tilde{s}_1, \tilde{s}_2)[n]$, for $n = 0, 1, \ldots, 4$, here we have shown the two signals for $n = 0, 1, \ldots, 8$. 
Notice that when $\vec{s}_1$ is periodic we don’t simply get the result from part a) repeated.